Solution Math 002-01 Quiz 7 (B)

Q.1: If A is a 3×3 matrix with $|A| = 2$ and B is a 4×4 matrix with $|B| = 3$, then find thevalue of the expression $\frac{1}{4}|3A| + \frac{2}{3}|2B|$.

Sol: $\frac{1}{4}|3A| + \frac{2}{3}|2B| = \frac{1}{4}3^3|A| + \frac{2}{3}2^4|B| = \frac{27}{4}(2) + \frac{32}{3}(3) = \frac{91}{2}$.

Q.2: Find value of the determinant $\begin{vmatrix} 1 & 2 & -1 & 3 \\ 3 & 1 & 0 & 2 \\ 2 & 4 & 1 & 1 \\ -3 & -6 & 6 & 2 \end{vmatrix}$.

Sol: $$\begin{vmatrix} 1 & 2 & -1 & 3 \\ 3 & 1 & 0 & 2 \\ 2 & 4 & 1 & 1 \\ -3 & -6 & 6 & 2 \end{vmatrix} \begin{matrix} -3R_1 + R_2 \\ -2R_1 + R_3 \\ 3R_1 + R_4 \\ \Rightarrow \end{matrix} \begin{vmatrix} 1 & 2 & -1 & 3 \\ 0 & -5 & 3 & -7 \\ 0 & 0 & 3 & -5 \\ 0 & 0 & 3 & 11 \end{vmatrix} \begin{matrix} -R_3 + R_4 \\ \Rightarrow \end{matrix} \begin{vmatrix} 1 & 2 & -1 & 3 \\ 0 & -5 & 3 & -7 \\ 0 & 0 & 3 & -5 \\ 0 & 0 & 0 & 16 \end{vmatrix}$$

$= (1)(-5)(3)(16) = -240.$

Q.3: If $A = \begin{bmatrix} 4 & 2 & 3 \\ 5 & 3 & 7 \\ 3 & 1 & 6 \end{bmatrix}$, find $C_{12} + M_{23} - C_{32}$.

Sol: $C_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 7 \\ 3 & 6 \end{vmatrix} = -9,\ M_{23} = \begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = -2,\ C_{32} = (-1)^{3+2}\begin{vmatrix} 4 & 3 \\ 5 & 7 \end{vmatrix} = -13$

$C_{12} + M_{23} - C_{32} = -9 - 2 + 13 = 2.$

Q.4: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -3 & -6 & -10 \end{bmatrix}$, find A^{-1} and show that $AA^{-1} = I$.

Sol: $$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -3 & -6 & -10 & 0 & 0 & 1 \end{array}\right] \begin{matrix} -2R_1 + R_2 \\ 3R_1 + R_3 \\ \Rightarrow \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 3 & 0 & 1 \end{array}\right]$$

$$\begin{matrix} R_3 + R_2 \\ 3R_3 + R_1 \\ \Rightarrow \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 10 & 0 & 3 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 3 & 0 & 1 \end{array}\right] \begin{matrix} -2R_2 + R_1 \\ -R_3 \\ \Rightarrow \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -2 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 & 0 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 8 & -2 & 1 \\ 1 & 1 & 1 \\ -3 & 0 & -1 \end{bmatrix} \text{ and } AA^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -3 & -6 & -10 \end{bmatrix}\begin{bmatrix} 8 & -2 & 1 \\ 1 & 1 & 1 \\ -3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$