Solution Math 002-01 Quiz 5 (A)

Q.1: Solve the linear system

$$\frac{2x-1}{2}+\frac{2y+1}{3}=1$$
$$\frac{2x+2}{4}+\frac{3y-1}{3}=1$$

Sol: $6\left(\frac{2x-1}{2}+\frac{2y+1}{3}=1\right) \Rightarrow 6x-3+4y+2=6 \quad 6x+4y=7$

$12\left(\frac{2x+2}{4}+\frac{3y-1}{3}=1\right) \Rightarrow 6x+6+12y-4=12 \quad 6x+12y=10$

$y=\frac{3}{8}$, and $x=\frac{7-4\left(\frac{3}{8}\right)}{6}=\frac{7-\frac{3}{2}}{6}=\frac{14-3}{12}=\frac{11}{12}$.

Q.2: Solve the nonlinear system

$$x^2-5xy-6y^2$$
$$x^2+2xy+y^2=2$$

Sol: $x^2-5xy-6y^2=0 \quad \Rightarrow (x+y)(x-6y)=0 \quad \Rightarrow x=-y$ or $x=6y$.

If $x=-y$, then $x^2+2xy+y^2=2 \quad \Rightarrow y^2-2y^2+y^2=0$ NO Solution

If $x=6y$, then $x^2+2xy+y^2=2 \quad \Rightarrow 36y^2+12y^2+y^2=2 \quad 49y^2=2 \quad \Rightarrow y=\pm\frac{\sqrt{2}}{7}$

$x=6y=\pm\frac{6\sqrt{2}}{7}$.

Thus the solutions are $\left(\frac{6\sqrt{3}}{7},\frac{\sqrt{2}}{7}\right)$ and $\left(-\frac{6\sqrt{3}}{7},-\frac{\sqrt{2}}{7}\right)$.

Q.3: Write the linear system in Augmented Matrix form and solve it by reducing into Echelon form,

$$\begin{array}{c} x-y+3z=10 \\ 2x-y+7z=24 \\ 3x-6y+7z=21 \end{array}.$$

Sol: $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 2 & -1 & 7 & 24 \\ 3 & -6 & 7 & 21 \end{array}\right] \overset{\substack{-2R_1+R_2 \\ -3R_1+R_3}}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 0 & 1 & 1 & 4 \\ 0 & -3 & -2 & -9 \end{array}\right] \overset{3R_2+R_3}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$z=3,\ y=1,\ x=2.$

The solution is $(2,1,3)$.

Q.4: If the augmented matrix of a system is $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & c^2 & 1 \\ 0 & 2 & 8 & c \end{array}\right]$, then find value of c for which the system is inconsistent.

Sol: $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & c^2 & 1 \\ 0 & 2 & 8 & c \end{array}\right] \overset{-2R_2+R_3}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & c^2 & 1 \\ 0 & 0 & 8-2c^2 & c-2 \end{array}\right]$

$8-2c^2=c-2 \quad \Rightarrow -2(c-2)(c+2)=c-2$

For inconsistant system, left side equal to zero and right side not equal to zero.

Thus $c=-2$.