Solution Math 002-01 Quiz 4 (B)

Q.1: Verify the identity $\dfrac{1+\cos x}{\sin x}+\dfrac{1}{\cot x+\csc x}=2\csc x$

$$LHS=\frac{1+\cos x}{\sin x}+\frac{1}{\cot x+\csc x}=\frac{1+\cos x}{\sin x}+\frac{1}{\frac{\cos x}{\sin x}+\frac{1}{\sin x}}=\frac{1+\cos x}{\sin x}+\frac{\sin x}{\cos x+1}=\frac{(1+\cos x)^2+\sin^2 x}{(1+\cos x)\sin x}$$

$$=\frac{1+\cos^2 x+2\cos x+\sin^2 x}{(1+\cos x)\sin x}=\frac{2+2\cos x}{(1+\cos x)\sin x}=\frac{2(1+\cos x)}{(1+\cos x)\sin x}=\frac{2}{\sin x}=2\csc x=RHS$$

Q.2: Find the exact value of $(i)\ \sin(195^o),\ (ii)\ \tan\left(\frac{13\pi}{12}\right)$

Sol: $\sin(195^\circ)=\sin(135^\circ+60^\circ)=\sin 135\cos 60+\cos 135\sin 60$

$$=\frac{\sqrt{2}}{2}\frac{1}{2}+\frac{-\sqrt{2}}{2}\frac{\sqrt{3}}{2}=\frac{\sqrt{2}\left(1-\sqrt{3}\right)}{4}.$$

$$\tan\left(\tfrac{13\pi}{12}\right)=\tan\left(\frac{9\pi+4\pi}{12}\right)=\tan\left(\frac{9\pi}{12}+\frac{4\pi}{12}\right)$$

$$=\tan\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)=\frac{\tan\left(\frac{3\pi}{4}\right)+\tan\left(\frac{\pi}{3}\right)}{1-\tan\left(\frac{3\pi}{4}\right)\cdot\tan\left(\frac{\pi}{3}\right)}=\frac{-1+\sqrt{3}}{1+\sqrt{3}}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}-1\right)^2}{3-1}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}.$$

Q.3: If $\sin x=-\dfrac{4}{5}$, x in Quadrant III and $\cos y=\dfrac{5}{13}$, y in Quadrant IV. Find $\tan(x+y)$.

Sol: $\cos x=-\sqrt{1-\sin^2 x}=-\sqrt{1-\dfrac{16}{25}}=-\dfrac{3}{5}$

$$\sin y=-\sqrt{1-\cos^2 y}=-\sqrt{1-\frac{25}{169}}=-\frac{12}{13}.$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{\frac{4}{3}-\frac{12}{5}}{1+\frac{4}{3}\frac{12}{5}}=\frac{\frac{20-36}{15}}{\frac{15+48}{15}}=-\frac{16}{63}$$

Q.4: Evaluate $\sin\left(\dfrac{3\pi}{8}\right)$ and $\cos(157.5^o)$

Sol: $$\sin\left(\frac{\frac{3\pi}{4}}{2}\right)=\sqrt{\frac{1-\cos\left(\frac{3\pi}{4}\right)}{2}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2+\sqrt{2}}}{2}.$$

$$\cos(157.5^\circ)=\cos\left(\frac{315^\circ}{2}\right)=-\sqrt{\frac{1+\cos(315^\circ)}{2}}=-\sqrt{\frac{1+\cos(45^\circ)}{2}}=-\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

$$=-\frac{\sqrt{2+\sqrt{2}}}{2}$$

Q.5: Verify the identity $\cos(3x)=4\cos^3 x-3\cos x$

Sol: $\cos(3x)=\cos(x+2x)=\cos x\cos(2x)-\sin x\sin(2x)$

$$=\cos x\left(2\cos^2 x-1\right)-\sin x(2\sin x\cos x)$$
$$=2\cos^3 x-\cos x-2\sin^2 x\cos x$$
$$=2\cos^3 x-\cos x-2\left(1-\cos^2 x\right)\cos x$$
$$=2\cos^3 x-\cos x-2\cos x+2\cos^3 x$$
$$=4\cos^3 x-3\cos x.$$