Maple Assignment #1

Chapter 10 "*Parametric Equations and Polar Coordinates*"

Name: **ID#:** **Sec#:**

```
> restart:
> with(plots):
Warning, the name changecoords has been redefined
```

Problem # 1: The parametric equations $f(t) = x1 + (x2 - x1)\, t$ and $g(t) = y1 + (y2 - y1)\, t$, where t in $[0, 1]$, describes the line segment from $P(x1, y1)$ to $Q(x2, y2)$. Use the follwoing three points P(-1,-1), Q(4,2), and R(1,5) to define three set of parametric equations such that the three line segments form a triangle. Plot the triangle and use animation to show how this triangle is being drawn.

NOTE: You can display three graphs using the following commands:
Plot1:=plot([f1(t),g1(t),t=0..1]):
similarly for Plot2 and Plot3 and then use display command to show the three graphs.

For animation, you can define three animation plots using
AnimPlot1:=animatecurve([f1(t),g1(t),t=0..1],frames=200):
similarly for AnimPlot2 and AnimPlot3 and then use display command to show the animation of the three graphs.

```
> x1:=-1;y1:=-1;x2:=4;y2:=2;x3:=1;y3:=5;
```

$$x1 := -1$$

$$y1 := -1$$

$$x2 := 4$$

$$y2 := 2$$

$$x3 := 1$$

$$y3 := 5$$

```
> f1(t):=x1+(x2-x1)*t;g1:=y1+(y2-y1)*t;
```

$$f1(t) := -1 + 5\, t$$

$$g1 := -1 + 3\, t$$

```
> f2(t):=x2+(x3-x2)*t;g2:=y2+(y3-y2)*t;
```

$$f2(t) := 4 - 3\, t$$

$$g2 := 2 + 3\, t$$

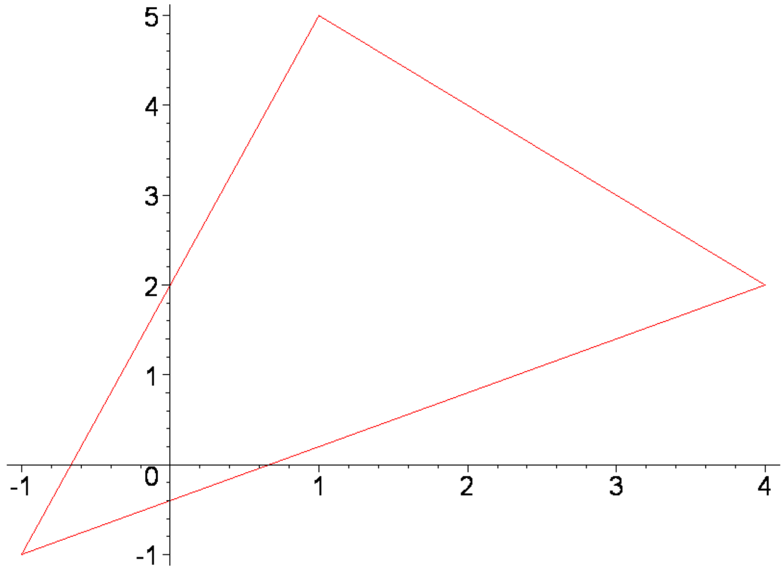
```
> f3(t):=x3+(x1-x3)*t;g3:=y3+(y1-y3)*t;
```

$$f3(t) := 1 - 2\, t$$

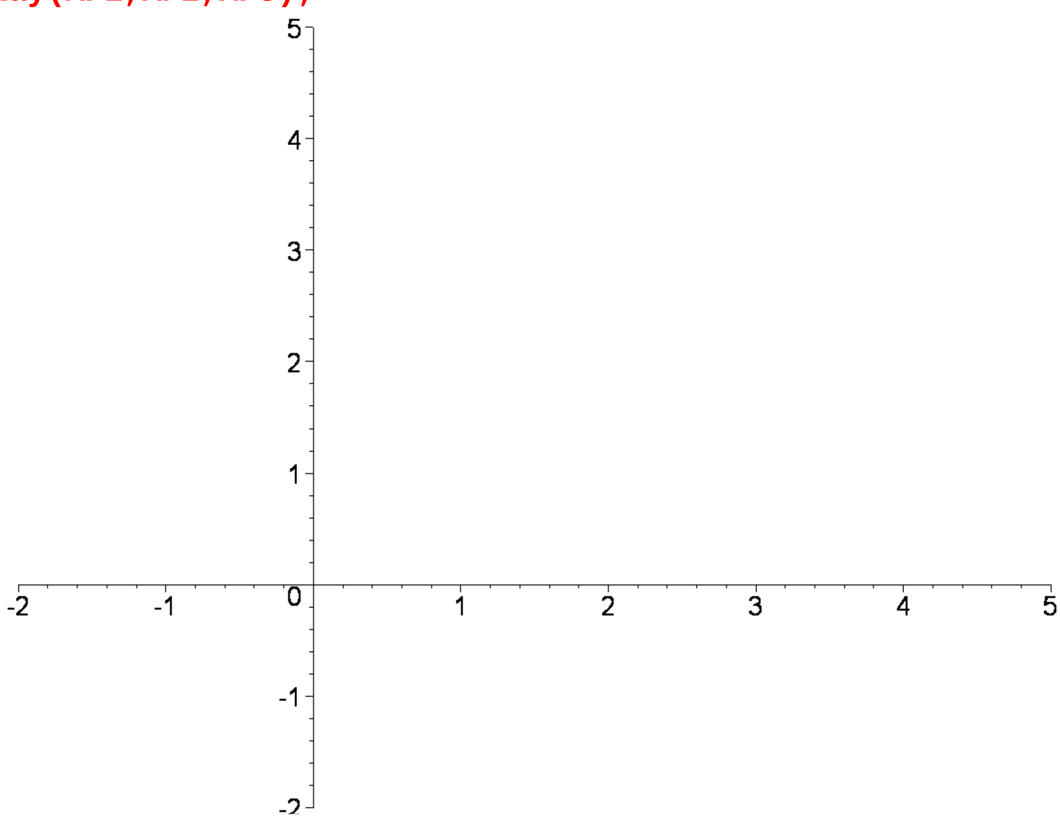
$$g3 := 5 - 6\, t$$

```
> P1:=plot([f1(t),g1(t),t=0..1]):
  P2:=plot([f2(t),g2(t),t=0..1]):
```

```
P3:=plot([f3(t),g3(t),t=0..1]):
> display(P1,P2,P3);
```



```
> AP1:=animatecurve([f1(t),g1(t),t=0..1],frames=200,view=[-2..5,-2..
5]):
AP2:=animatecurve([f2(t),g2(t),t=0..1],frames=200,view=[-2..5,-2..
5]):
AP3:=animatecurve([f3(t),g3(t),t=0..1],frames=200,view=[-2..5,-2..
5]):
> display(AP1,AP2,AP3);
```



>

Problem # 2: Find parametric equations for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, use these equations to plot the ellipse for a = 4, and b = 1, b = 2, b = 4, b = 6, and b = 8. Explain how does the shape of the ellipse varies as value of b changes. Plot these graphs in different windows. Size of each window should not be more than 3 by 3 inches.

```
> a:=4;b:=1;
```

$$a := 4$$

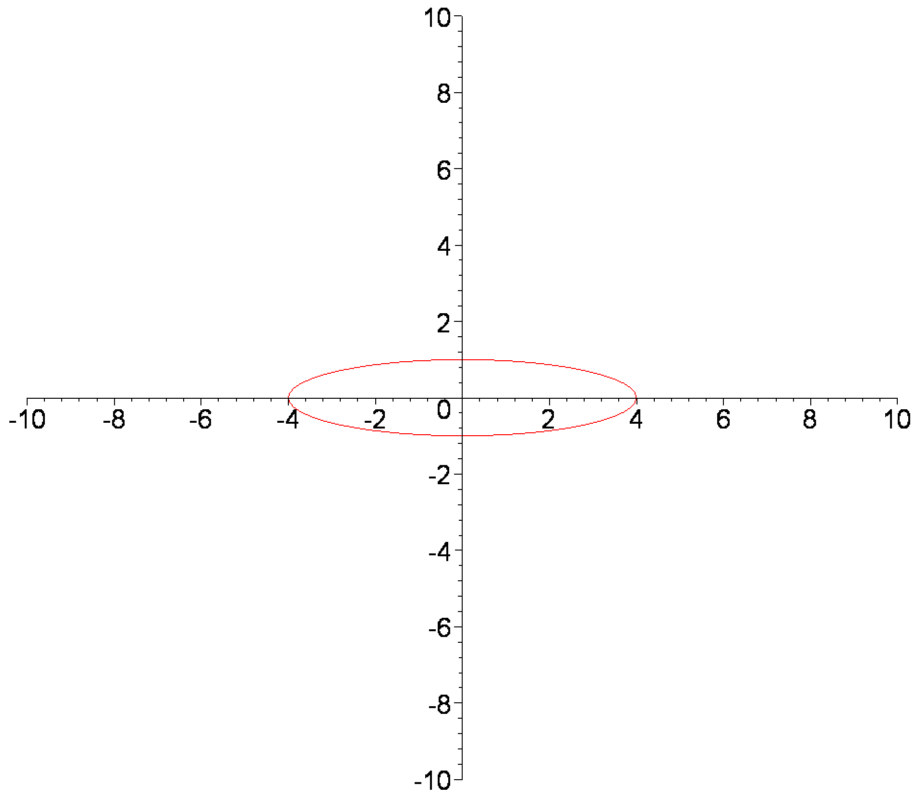
$$b := 1$$

```
> x(t):=a*cos(t);y(t):=b*sin(t);
```

$$x(t) := 4\cos(t)$$

$$y(t) := \sin(t)$$

```
> plot([x(t),y(t),t=0..2*Pi],view=[-10..10,-10..10]);
```




```
> a:=4;b:=2;
```

$$a := 4$$

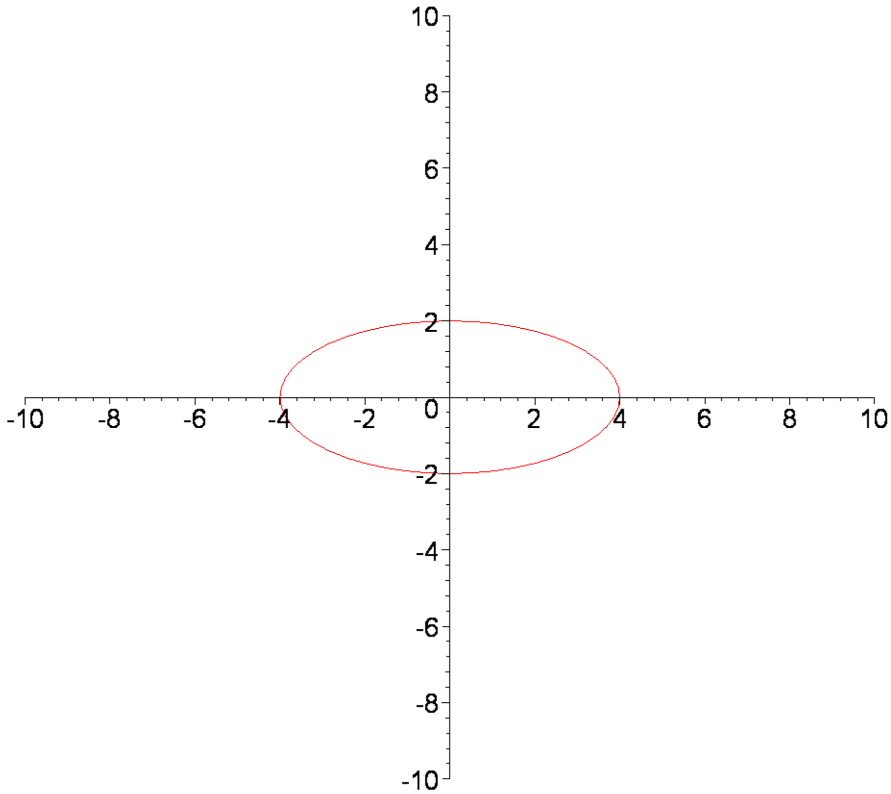
$$b := 2$$

```
> x(t):=a*cos(t);y(t):=b*sin(t);
```

$$x(t) := 4\cos(t)$$

$$y(t) := 2\sin(t)$$

```
> plot([x(t),y(t),t=0..2*Pi],view=[-10..10,-10..10]);
```




```
> a:=4;b:=4;
```

$$a := 4$$

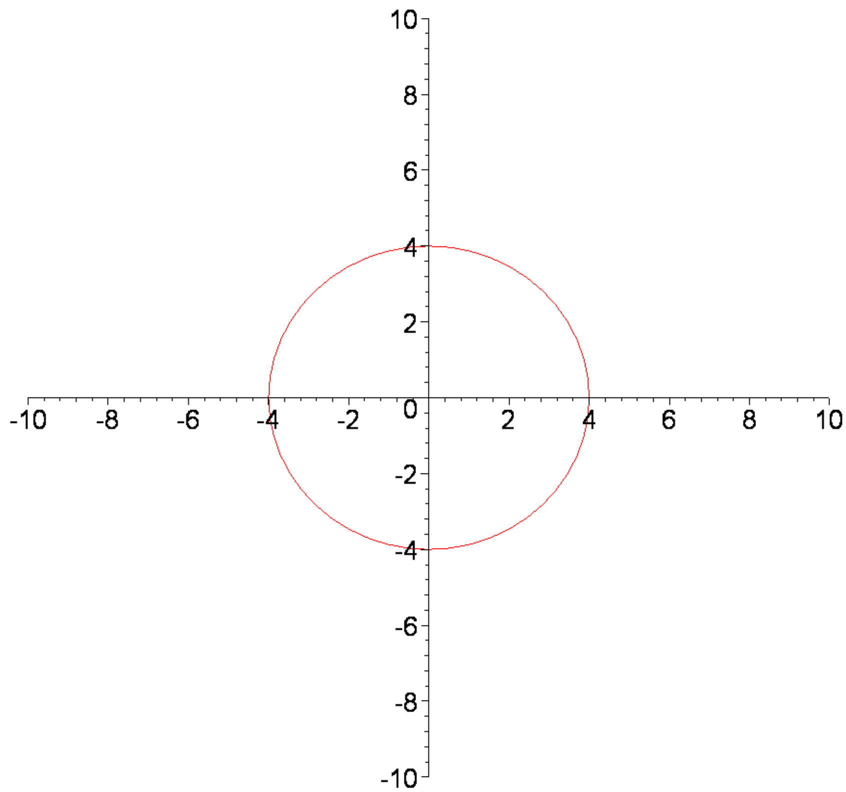
$$b := 4$$

```
> x(t):=a*cos(t);y(t):=b*sin(t);
```

$$\mathrm{x}(t) := 4\cos(t)$$

$$\mathrm{y}(t) := 4\sin(t)$$

```
> plot([x(t),y(t),t=0..2*Pi],view=[-10..10,-10..10]);
```




```
> a:=4;b:=6;
```

$$a := 4$$

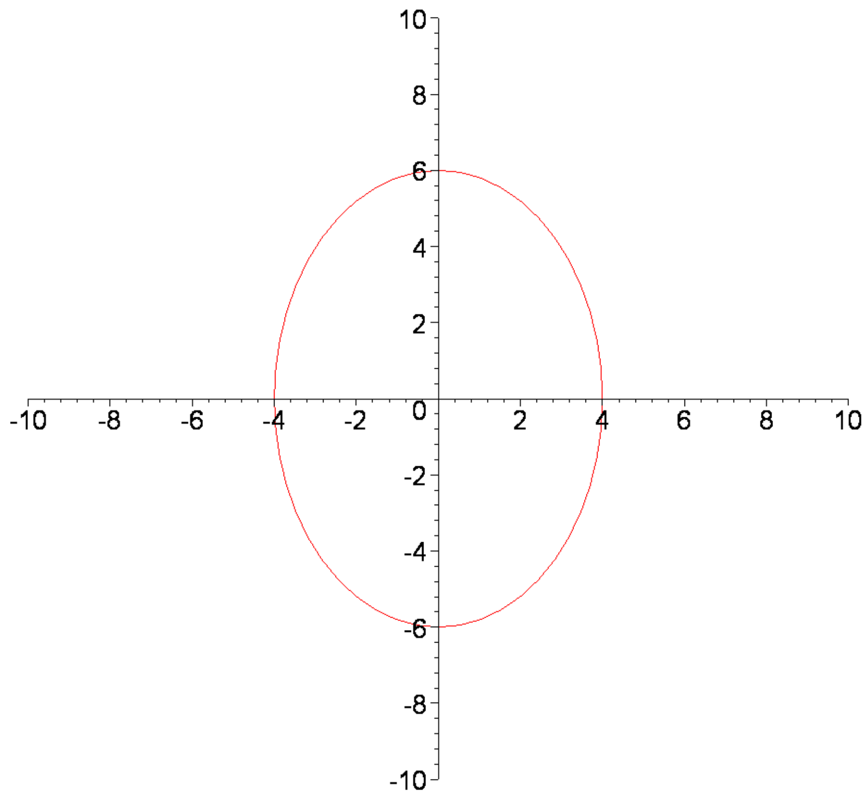
$$b := 6$$

```
> x(t):=a*cos(t);y(t):=b*sin(t);
```

$$x(t) := 4\cos(t)$$

$$y(t) := 6\sin(t)$$

```
> plot([x(t),y(t),t=0..2*Pi],view=[-10..10,-10..10]);
```




```
> a:=4;b:=8;
```

$$a := 4$$

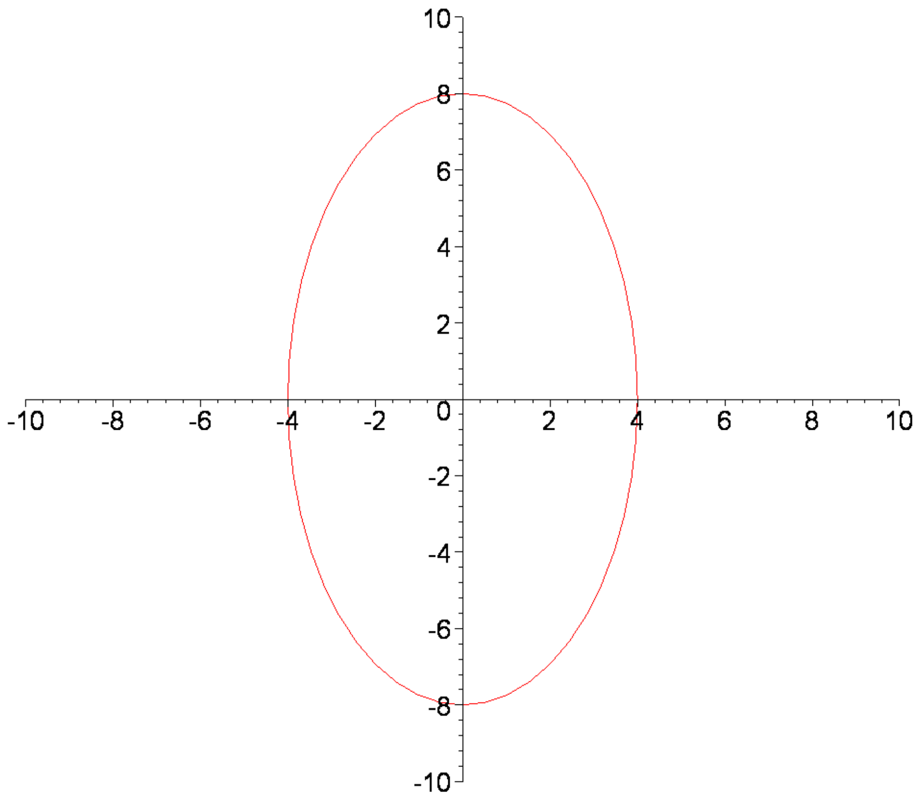
$$b := 8$$

```
> x(t):=a*cos(t);y(t):=b*sin(t);
```

$$\mathrm{x}(t) := 4\cos(t)$$

$$\mathrm{y}(t) := 8\sin(t)$$

```
> plot([x(t),y(t),t=0..2*Pi],view=[-10..10,-10..10]);
```



>

Problem # 3: Suppose position of one particle is given by $x1 = a\sin(t)$ $y1 = b\cos(t)$, t in $[0, 2\pi]$ and the position of a second particle is given by $x2 = -3 + \cos(t)$ $y2 = 1 + \sin(t)$, t in $[0, 2\pi]$. Plot these two graphs in the same window for *a* = 3 and *b* = 2. Write how many points of ntersections are there? What difference do you see if *a* = 2 and *b* = 3. Animate these two curves and find if there is any collision point, a point where two graphs intersect for the same value of *t*. For plotting and Animating two graps in the same window, use technique given in the NOTE above.

```
> a:=3; b:=2;
```

$$a := 3$$

$$b := 2$$

```
> x1:=a*sin(t);y1:=b*cos(t);
```

$$x1 := 3\sin(t)$$

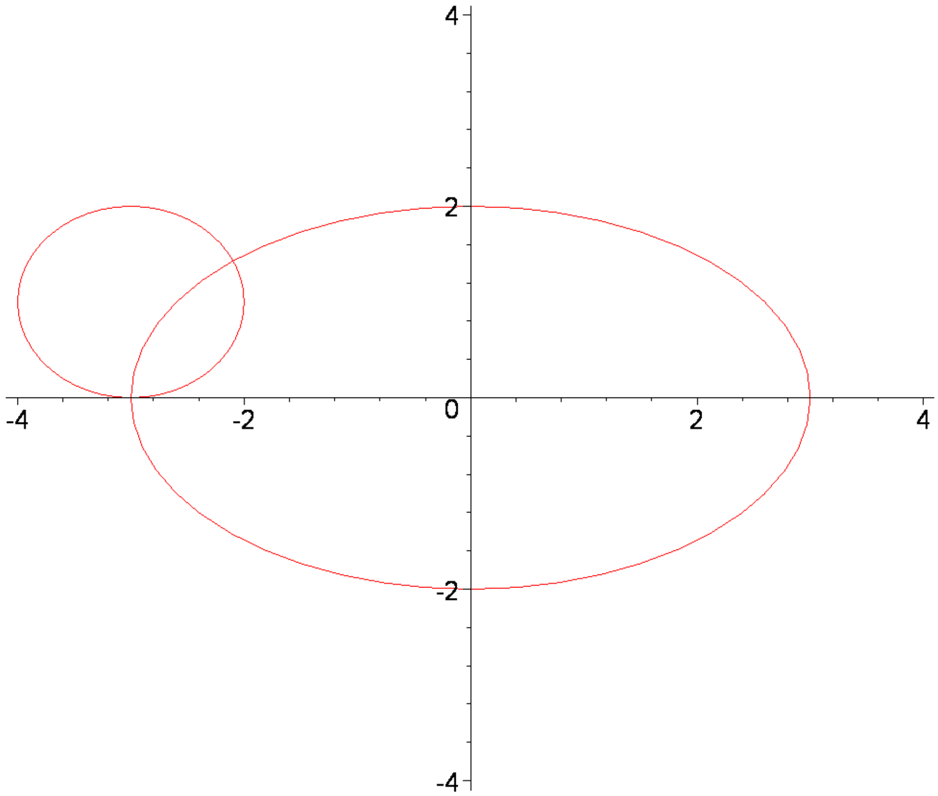
$$y1 := 2\cos(t)$$

```
> x2:=-3+cos(t);y2:=1+sin(t);
```

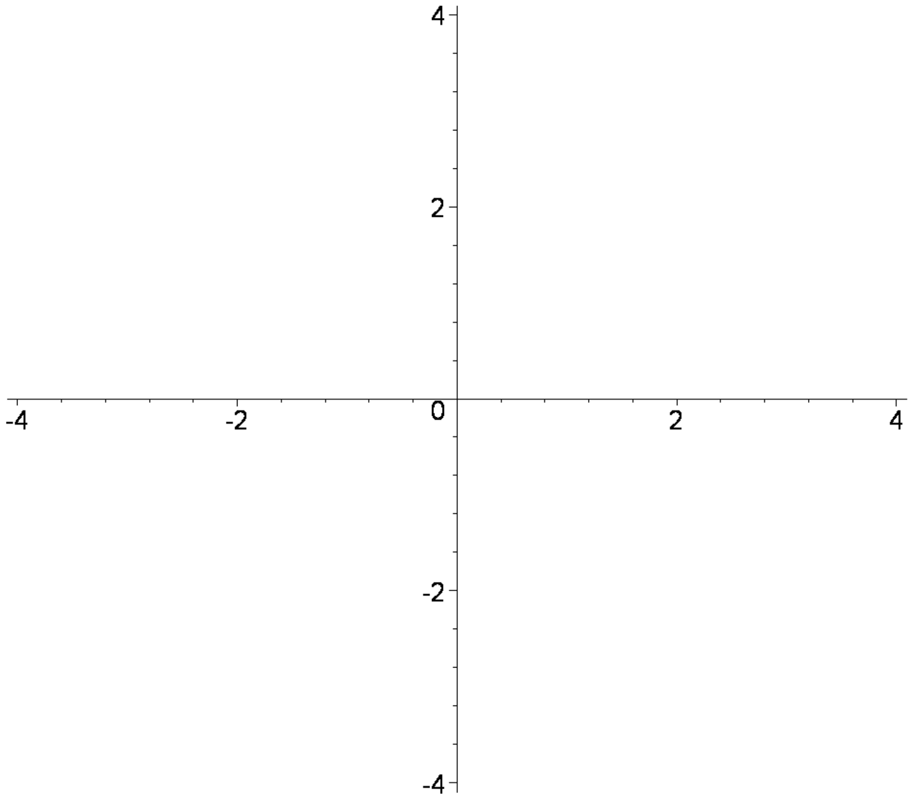
$$x2 := -3 + \cos(t)$$

$$y2 := 1 + \sin(t)$$

```
> P1:=plot([x1(t),y1(t),t=0..2*Pi]):
  P2:=plot([x2(t),y2(t),t=0..2*Pi]):
> display(P1,P2,view=[-4.1..4.1,-4.1..4.1]);
```




```
> AP1:=animatecurve([x1(t),y1(t),t=0..2*Pi],frames=200,view=[-4.1..4
  .1,-4.1..4.1]):
  AP2:=animatecurve([x2(t),y2(t),t=0..2*Pi],frames=200,view=[-4.1..4
  .1,-4.1..4.1]):
> display(AP1,AP2);
```




```
> a:=2; b:=3;
```

$$a := 2$$

$$b := 3$$

```
> x1:=a*sin(t);y1:=b*cos(t);
```

$$x1 := 2\sin(t)$$

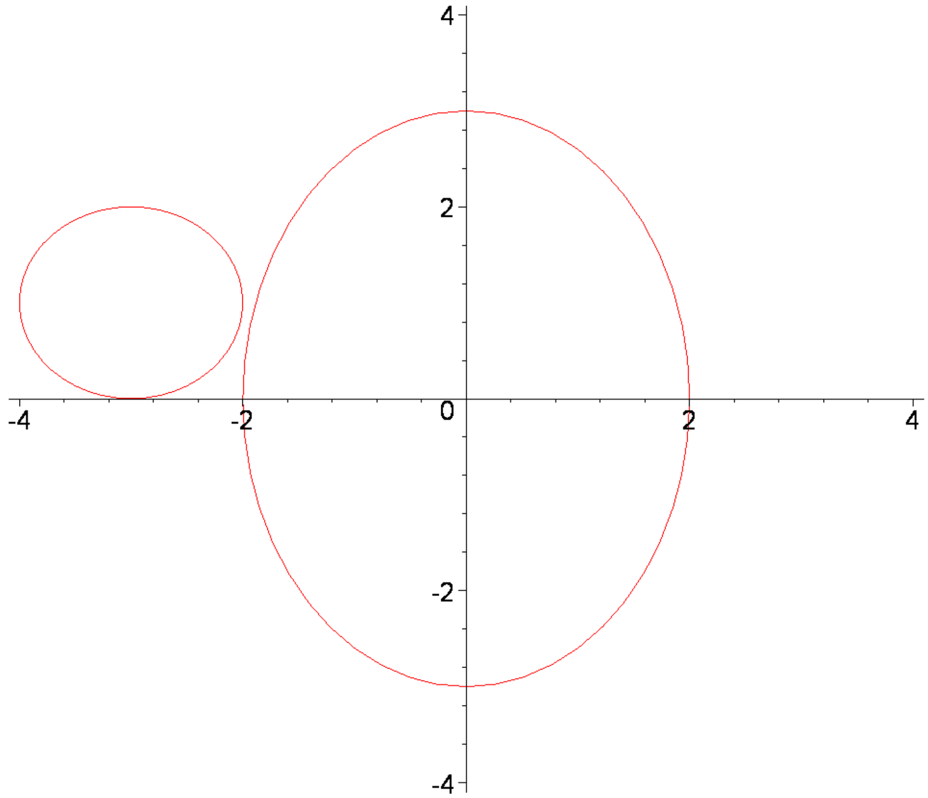
$$y1 := 3\cos(t)$$

```
> x2:=-3+cos(t);y2:=1+sin(t);
```

$$x2 := -3 + \cos(t)$$

$$y2 := 1 + \sin(t)$$

```
> P1:=plot([x1(t),y1(t),t=0..2*Pi]):
  P2:=plot([x2(t),y2(t),t=0..2*Pi]):
> display(P1,P2,view=[-4.1..4.1,-4.1..4.1]);
```



>

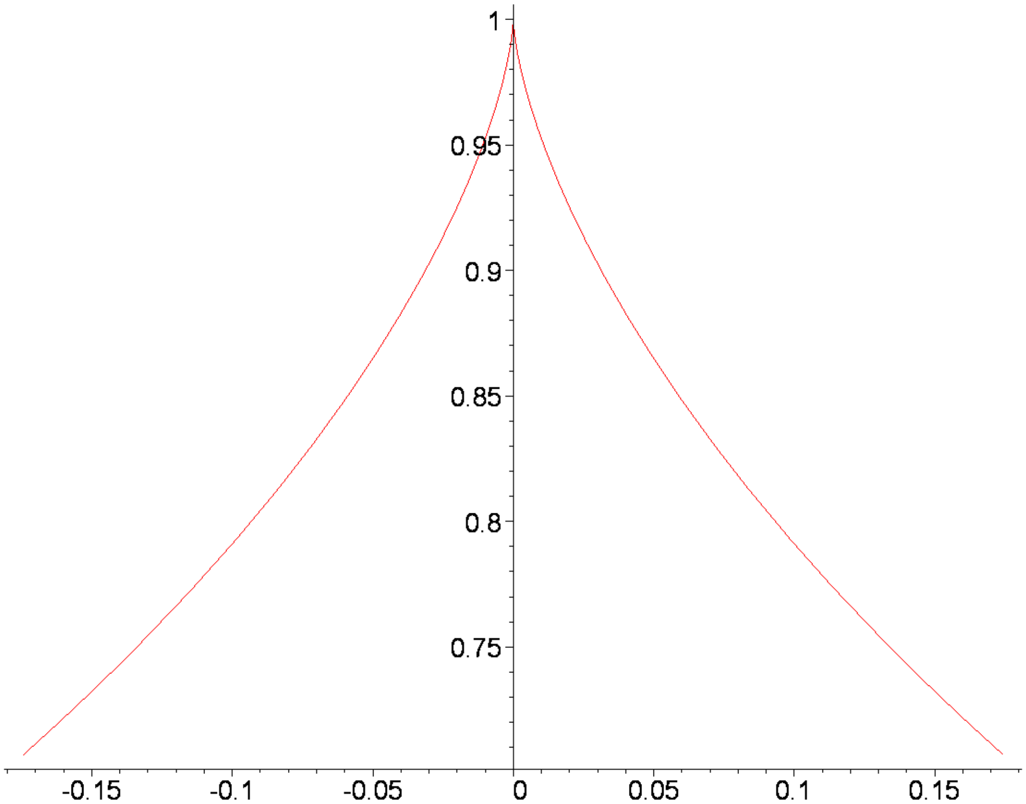
Problem # 4: Sketch graph of the curve $x(t) = \cos(t) + \log\left(\tan\left(\frac{t}{2}\right)\right)$ $y(t) = \sin(t)$, t in $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$. Also find its length. (If you get a strange answer, use evalf(%); command to get its numerical value.)

```
> x(t):=cos(t)+log(tan(t/2)); y(t):=sin(t);
```

$$x(t) := \cos(t) + \ln\left(\tan\left(\frac{1}{2}t\right)\right)$$

$$y(t) := \sin(t)$$

```
> plot([x(t),y(t),t=Pi/4..3*Pi/4]);
```




```
> L:=int(sqrt(diff(x(t),t)^2+diff(y(t),t)^2),t=Pi/4..3*Pi/4):
> evalf(%);
```

$$0.6931471785 + 0.\, I$$

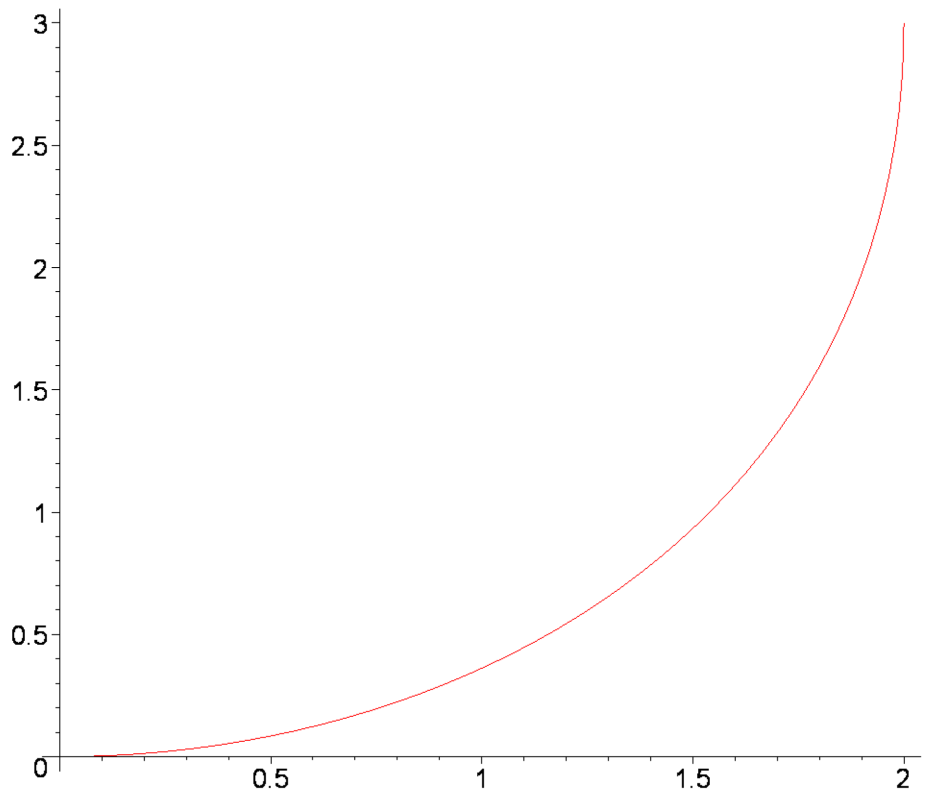
```
>
```

Problem # 5: Find area of surface of revolution obtained by rotating the curve $x(t) = 3\,t - t^3$, $y(t) = 3\,t^2$, t in $[0, 1]$.

```
> x(t):=3*t-t^3; y(t):=3*t^2;
```

$$x(t) := 3\,t - t^3$$
$$y(t) := 3\,t^2$$

```
> plot([x(t),y(t),t=0..1]);
```




```
> A:=2*Pi*int(y(t)*sqrt(diff(x(t),t)^2+diff(y(t),t)^2),t=0..1);
```

$$A := \frac{48}{5}\pi$$

```
> evalf(%);
```

$$30.15928948$$

```
>
```

Problem # 6: Find area of surface of revolution obtained by rotating the curve $x(t) = t + t^3$, $y(t) = t - \frac{1}{t^2}$, t in $[0, 2]$.(If you get a strange answer, use evalf(%); command to get its numerical value.)

```
> x(t):=t+t^3; y(t):=t-1/t^2;
```

$$x(t) := t + t^3$$

$$y(t) := t - \frac{1}{t^2}$$

```
> surfarea:=2*Pi*int(y(t)*sqrt(diff(x(t),t)^2+diff(y(t),t)^2),t=0..2
);
```

$$surfarea := 2\pi \int_0^2 \left(t - \frac{1}{t^2}\right)\sqrt{(1+3t^2)^2 + \left(1 + \frac{2}{t^3}\right)^2}\, dt$$

```
> evalf(%);
```

$$\text{Float}(-\infty)$$

Problem # 7: Plot the two polar curves $r1 = 1 + \sin(t)$, and $r2 = 1 + \sin\left(t - \frac{\pi}{3}\right)$ in the same window. Write how first graph is related to the second graph.

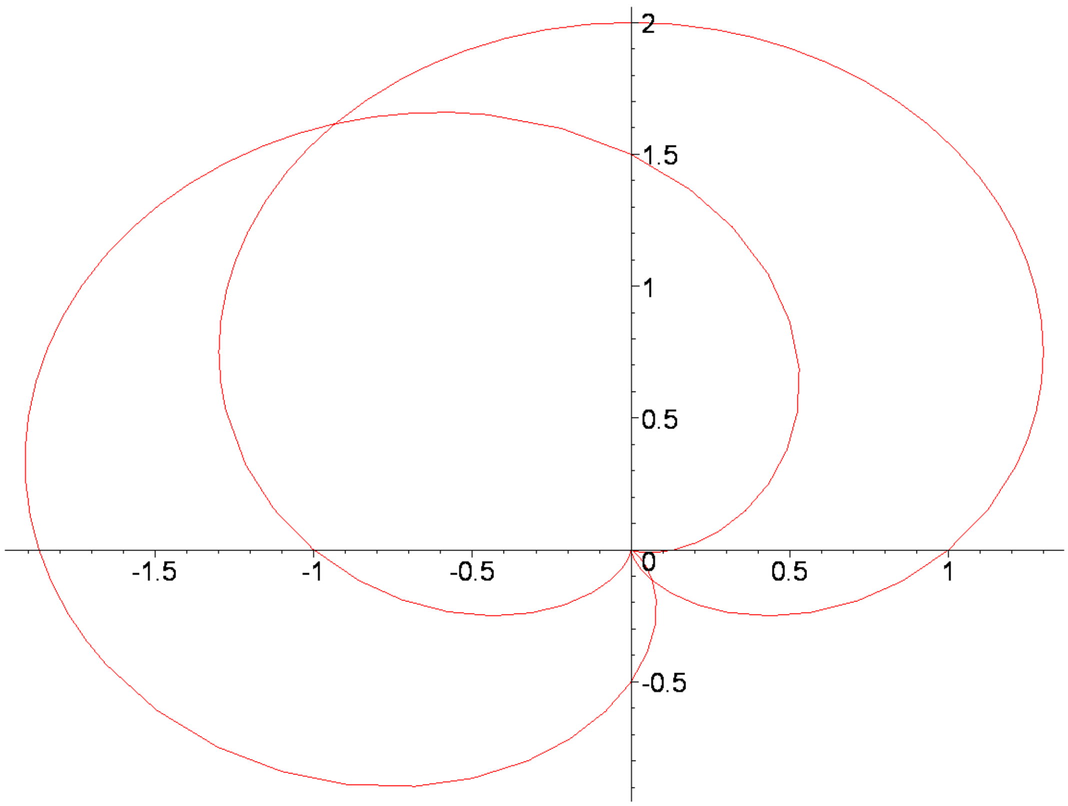
```
> r1(t):=1+sin(t); r2(t):=1+sin(t-Pi/3);
```

$$\text{r1}(t) := 1 + \sin(t)$$

$$\text{r2}(t) := 1 - \cos\left(t + \frac{1}{6}\pi\right)$$

```
> P1:=plot([r1(t), t, t=0..2*Pi],coords=polar):
  P2:=plot([r2(t), t, t=0..2*Pi],coords=polar):
```

```
> display(P1,P2);
```



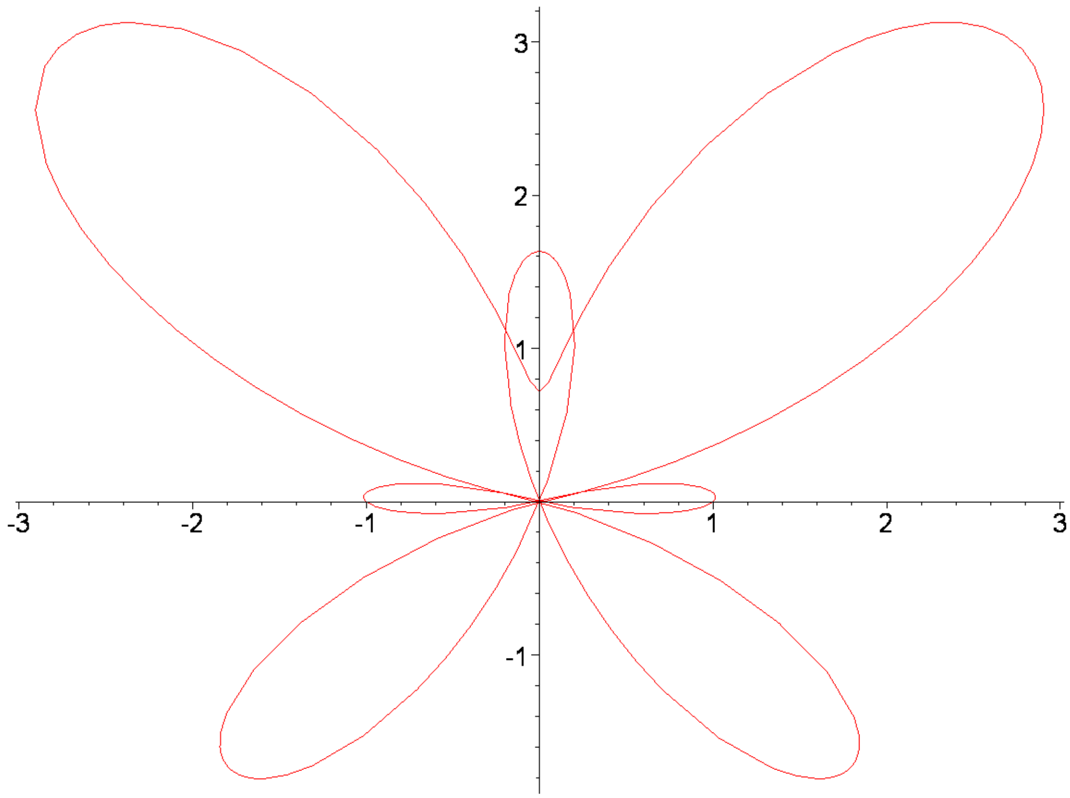
Graph of second function is rotated 60^o counter clockwise.

Problem # 8: Plot the polar curves $r = \mathbf{e}^{\sin(t)} - 2\cos(4t)$.

```
> r(t) := exp(sin(t))-2*cos(4*t);
```

$$\text{r}(t) := \mathbf{e}^{\sin(t)} - 2\cos(4t)$$

```
> plot([r(t), t, t=0..2*Pi],coords=polar);
```



Problem # 9: Plot the two polar curves $r1 = 1 + \cos(t)$, and $r2 = 3\cos(t)$ in the same window. Find point of intersection of the two curves using "solve(r1(t)-r2(t)=0,t)" and find the area that lies inside both curves.

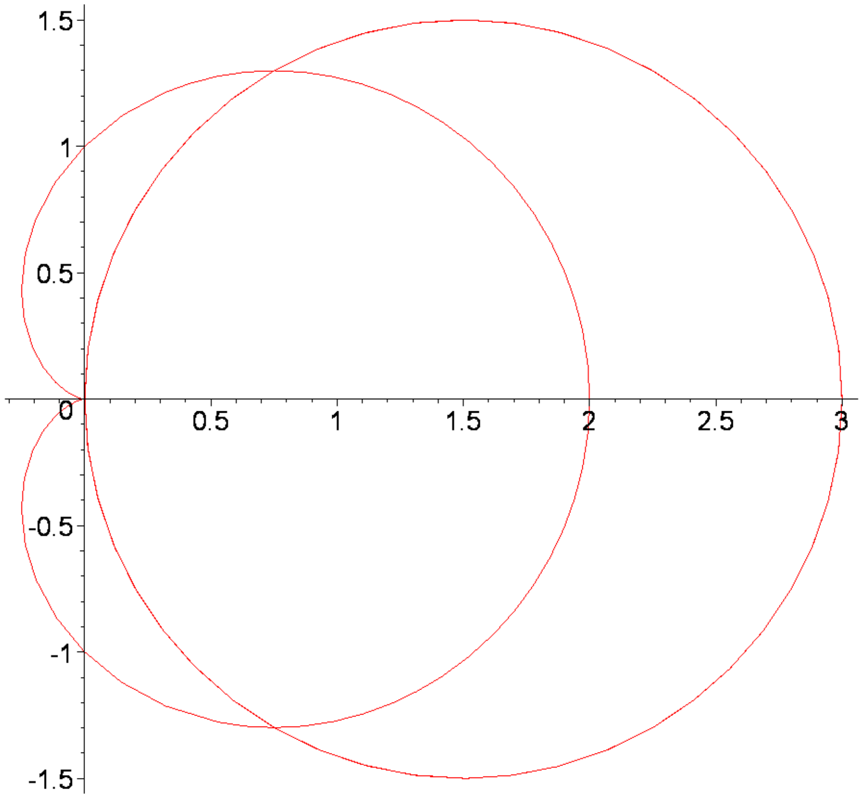
```
> r1(t):=1+cos(t); r2(t):=3*cos(t);
```

$$r1(t) := 1 + \cos(t)$$

$$r2(t) := 3\cos(t)$$

```
> P1:=plot([r1(t), t, t=0..2*Pi],coords=polar):
  P2:=plot([r2(t), t, t=0..2*Pi],coords=polar):
```

```
> display(P1,P2);
```




```
> solve(r1(t)-r2(t)=0,t);
```

$$\frac{1}{3}\pi$$

Problem # 10: Plot the two polar curves $r1 = \sin(2\,t)$, and $r2 = \sin(t)$ in the same window. Find point of intersection of the two curves using "solve(r1(t)-r2(t)=0,t)" and find the area that lies inside both curves.

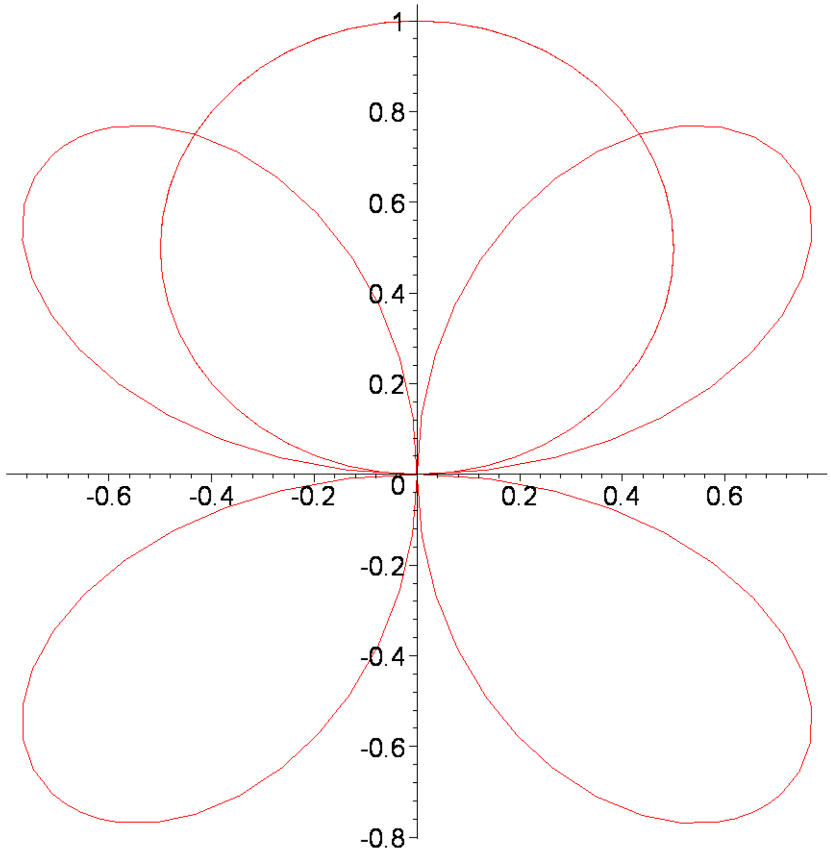
```
> r1(t):=sin(2*t); r2(t):=sin(t);
```

$$r1(t) := \sin(2\,t)$$

$$r2(t) := \sin(t)$$

```
> P1:=plot([r1(t), t, t=0..2*Pi],coords=polar):
  P2:=plot([r2(t), t, t=0..2*Pi],coords=polar):
```

```
> display(P1,P2);
```




```
> solve(r2(t)-r1(t)=0,t);
```

$$\pi, 0, \frac{1}{3}\pi, -\frac{1}{3}\pi$$

```
>
```