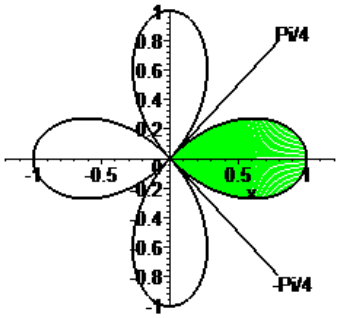
Area Bounded by Polar Curves

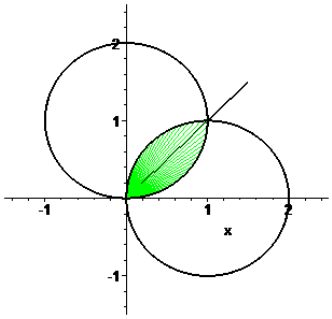
1. Area bounded by one loop of $r = \cos(2\,t)$

$$A := \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(2\,t)^2\,dt = \frac{\pi}{8}$$



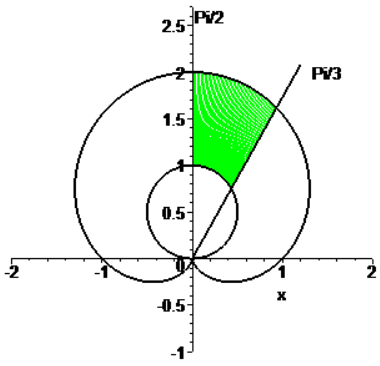
2. Area bounded by $r = 4\cos(t)$ and $r = 4\sin(t)$

$$A := \int_{0}^{\frac{\pi}{4}} 16\,\sin(t)^2\,dt = -4 + 2\,\pi$$



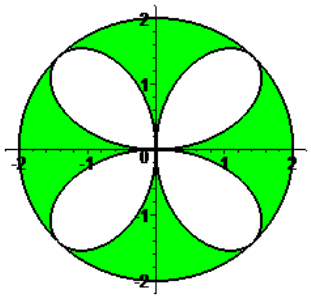
3. Area bounded by inside $r = 1 + \sin(t)$ and outside $r = \sin(t)$

$$A := \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 + \sin(t))^2 - \sin(t)^2\,dt = \frac{\pi}{12} + \frac{1}{2}$$



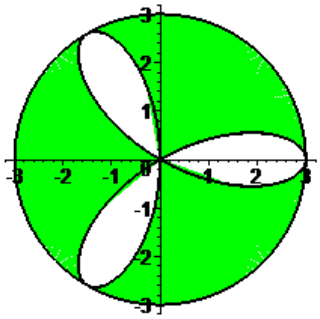
4. Area outside $r = 2\sin(2t)$ and inside $r = 2$

$$A := 4\pi - 2\int_0^{\frac{\pi}{2}} 4\sin(2t)^2\,dt = 2\pi$$



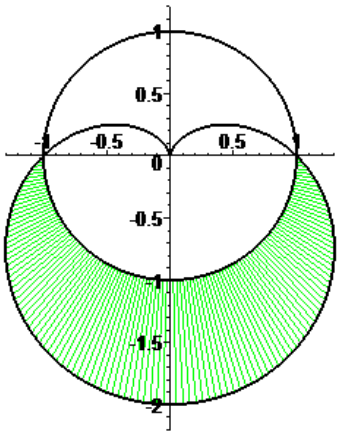
5. Area outside $r = 3\cos(3t)$ and inside $r = 3$

$$A := 9\pi - \frac{3}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 9\cos(3t)^2\,dt = \frac{27\pi}{4}$$



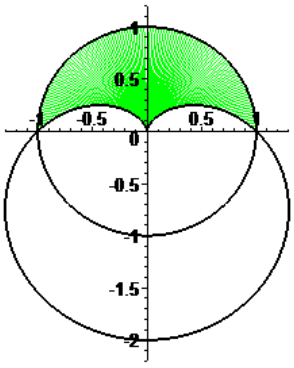
6. Area inside cardioid $r = 1 - \sin(t)$ and outside $r = 1$

$$A := \int_{-\frac{\pi}{2}}^{0} (1-\sin(t))^2 - 1\,dt = \frac{\pi}{4} + 2$$



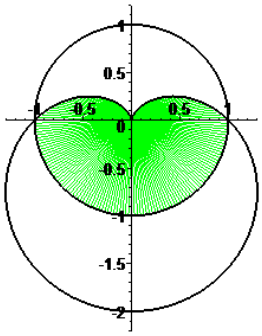
7. Area outside cardioid $r = 1 - \sin(t)$ and inside $r = 1$

$$A := \int_0^{\frac{\pi}{2}} 1 - (1 - \sin(t))^2 \, dt = 2 - \frac{\pi}{4}$$



8. Area inside both cardioid $r = 1 - \sin(t)$ and $r = 1$

$$A := \int_{-\frac{\pi}{2}}^{0} 1 \, dt + \int_0^{\frac{\pi}{2}} (1 - \sin(t))^2 \, dt = \frac{5\,\pi}{4} - 2$$



9. Area inside both $r = \sin(2\,t)$ and $r = \cos(2\,t)$

$$A := 8 \int_0^{\frac{\pi}{8}} \sin(t)^2 \, dt = -4 \cos\left(\frac{3\,\pi}{8}\right) \sin\left(\frac{3\,\pi}{8}\right) + \frac{\pi}{2}$$