Solution Math 201 Quiz 4 (B)

Q.1: Show that $u(x,t) = \ln(x+at) + \cos(x-at)$ is a solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

Sol: $\frac{\partial u}{\partial t} = \frac{a}{x+at} + a\sin(x-at)$ and $\frac{\partial^2 u}{\partial t^2} = \frac{-a^2}{(x+at)^2} - a^2\cos(x-at)$

$\frac{\partial u}{\partial x} = \frac{1}{x+at} - \sin(x-at)$ and $\frac{\partial^2 u}{\partial x^2} = \frac{-1}{(x+at)^2} - \cos(x-at)$

Thus $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

Q.2: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function $F(x,y,z) = x^3 + y^2 + z + 3xy - 2xz + 4yz + 6$.

Sol: $\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{3x^2+3y-2z}{1-2x+4y}$

$\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{2y+3x+4z}{1-2x+4y}$.

Q.3: Show that equation of the tangent plane to the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point $P_o(x_o, y_o, z_o)$ is $\frac{xx_o}{a^2} + \frac{yy_o}{b^2} + \frac{zz_o}{c^2} = 1$.

Sol: $\nabla f(x_o, y_o, z_o) = < \frac{2x_o}{a^2}, \frac{2y_o}{b^2}, \frac{2z_o}{c^2} >$ and equation of the tangent plane is

$\frac{2x_o}{a^2}(x - x_o) + \frac{2y_o}{b^2}(y - y_o) + \frac{2z_o}{c^2}(z - z_o) = 0$

$\frac{2x\ x_o}{a^2} + \frac{2y\ y_o}{b^2} + \frac{2zz_o}{c^2} = \frac{2x_o^2}{a^2} + \frac{2y_o^2}{b^2} + \frac{2z_o^2}{c^2} = 2(1)$

$\frac{x\ x_o}{a^2} + \frac{y\ y_o}{b^2} + \frac{zz_o}{c^2} = 1$