Solution Math 201 Quiz 4 (A)

Q.1: Show that $u(x,t) = \sin(x - at) + \cos(x + at)$ is a solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

Sol: $\frac{\partial u}{\partial t} = -a\cos(x - at) - a\sin(x + at)$ and $\frac{\partial^2 u}{\partial t^2} = -a^2\sin(x - at) - a^2\cos(x + at)$

$\frac{\partial u}{\partial x} = \cos(x - at) - \sin(x + at)$ and $\frac{\partial^2 u}{\partial x^2} = -\sin(x - at) - \cos(x + at)$

Thus $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

Q.2: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function $F(x,y,z) = x + y^2 + z^3 - 2xy + 3xz - 5yz + 6$.

Sol:

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{1 - 2y + 3z}{3z^2 + 3x - 5y}$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{2y - 2x - 5z}{3z^2 + 3x - 5y}.$$

Q.3: Show that equation of the tangent plane to the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 0$ at the point $P_o(x_o, y_o, z_o)$ is $\frac{2xx_o}{a^2} + \frac{2yy_o}{b^2} = \frac{z + z_o}{c}$.

Sol: $\nabla f(x_o, y_o, z_o) = < \frac{2x_o}{a^2}, \frac{2y_o}{b^2}, \frac{-1}{c} >$ and equation of the tangent plane is

$$\frac{2x_o}{a^2}(x - x_o) + \frac{2y_o}{b^2}(y - y_o) - \frac{1}{c}(z - z_o) = 0$$

$$\frac{2x\ x_o}{a^2} + \frac{2y\ y_o}{b^2} - \frac{z}{c} = \frac{2x_o^2}{a^2} + \frac{2y_o^2}{b^2} - \frac{z_o}{c} = 2\left[\frac{x_o^2}{a^2} + \frac{y_o^2}{b^2}\right] - \frac{z_o}{c} = \frac{2z_o}{c} - \frac{z_o}{c}$$

$$\frac{2x\ x_o}{a^2} + \frac{2y\ y_o}{b^2} = \frac{z}{c} + \frac{z_o}{c} = \frac{z + z_o}{c}.$$