

Q.1: Reduce the equation and classify the surface, $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$.

Sol: $x^2 - 4x - y^2 - 2y + z^2 - 2z = -4$
 $x^2 - 4x + 4 - y^2 - 2y - 1 + z^2 - 2z + 1 = -4 + 4 - 1 + 1$
 $(x - 2)^2 - (y + 1)^2 + (z - 1)^2 = 0$

The surface is a cone.

Q.2: Identify the surface $\rho^2 (\sin^2 \phi - 3 \cos^2 \phi) = 3$.

Sol: $\rho^2 \sin^2 \phi - 3\rho^2 \cos^2 \phi = 3$
 $r^2 - 3z^2 = 3$
 $x^2 + y^2 - 3z^2 = 3$

The surface is hyperboloid of one sheet.

Q.3: Find the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2}$ if exist, or show that limit does not exist.

Sol: Let $x = 0$ and $y \rightarrow 0$, then $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = \lim_{y \rightarrow 0} \frac{0 \sin^2 y}{2y^2} = 0$.

Let $y = 0$ and $x \rightarrow 0$, then $\lim_{x \rightarrow 0} \frac{x^2 \sin^2 0}{x^2} = 0$.

We suspect that limit may exist.

$$0 \leq x^2 \leq x^2 + 2y^2$$

$$0 \leq \frac{x^2}{x^2 + 2y^2} \leq 1$$

$$0 \leq \frac{x^2 \sin^2 y}{x^2 + 2y^2} \leq \sin^2 y$$

since $\lim_{y \rightarrow 0} \sin^2 y = 0$, therefore by sandwich theorem, $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0$.