Math 201
Maple Handout # 10.4
Area and Arc Lengths in Polar Coordinates

In this assignment we will learn how to find area enclosed by polar curves.
Whenever you open a Maple file, press ENTER with cursor anywhere on **restart:** and on **with(plots):**
NOTE: To type click on T icon. To insert > for typing math, click on [> icon

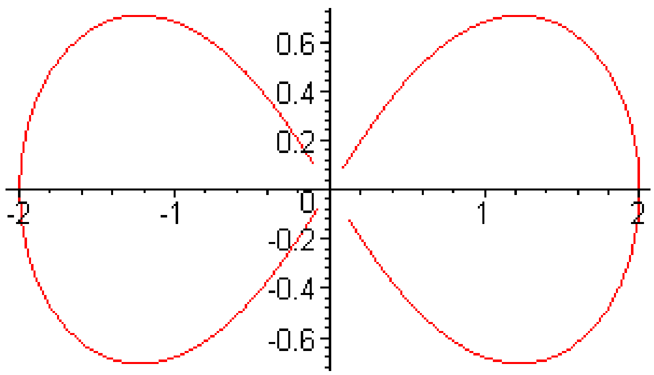
```
> restart:
> with(plots):
Warning, the name changecoords has been redefined
```

Problem # 11

```
> r(t):=sqrt(4*cos(2*t));
```

$$r(t) := 2\sqrt{\cos(2t)}$$

```
> plot([r(t),t,t=0..2*Pi],coords=polar);
```



We evaluate the integral

$$2\int_0^{\frac{\pi}{4}} r(t)^2\, dt$$

```
> 4*1/2*int(r(t)^2,t=0..Pi/4);
```

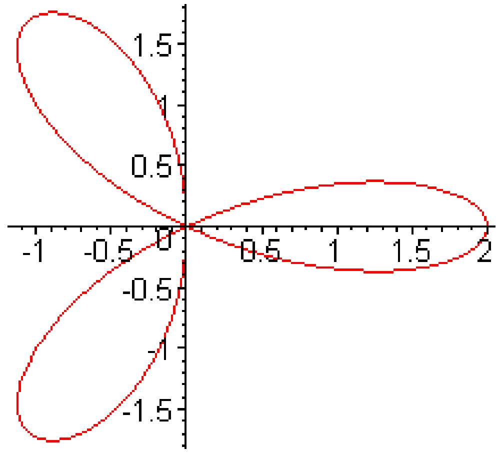
$$4$$

Problem # 13

```
> r1(t):=2*cos(3*t);
```

$$r1(t) := 2\cos(3t)$$

```
> plot([r1(t),t,t=0..Pi],coords=polar);
```



We evaluate the integral

$$3\int_0^{\frac{\pi}{6}} \mathrm{r1}(t)^2\,dt$$

```
> 1/2*6*int(r1(t)^2,t=0..Pi/6);
```

$$-\frac{39}{2}\sqrt{3}+\frac{11}{2}\pi+36$$

Problem # 29

```
> r1(t):=sin(t); r2(t):=cos(t);
```

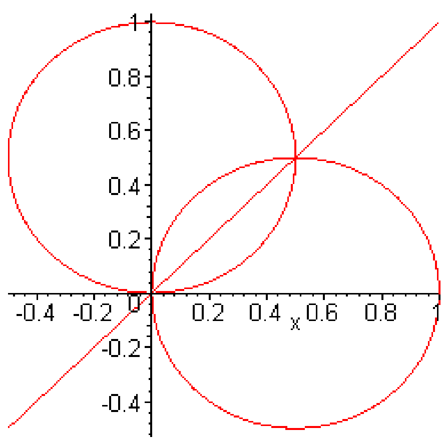
$$\mathrm{r1}(t) := \sin(t)$$

$$\mathrm{r2}(t) := \cos(t)$$

```
> P1:=plot([r1(t),t,t=0..2*Pi],coords=polar):
```

```
> P2:=plot([r2(t),t,t=0..2*Pi],coords=polar):
```

```
> P3:=plot(x,x=-.5..1):
> display(P1,P2,P3);
```



We will evaluate the integral

$$.5\int_{0}^{\frac{\pi}{4}} \sin(t)^2\,dt + .5\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(t)^2\,dt$$

```
> 0.5*(int(sin(t)^2,t=0..Pi/4))+0.5*(int(cos(t)^2,t=Pi/4..Pi/2));
```

$$-.2500000000+0.1250000000\pi$$

\>

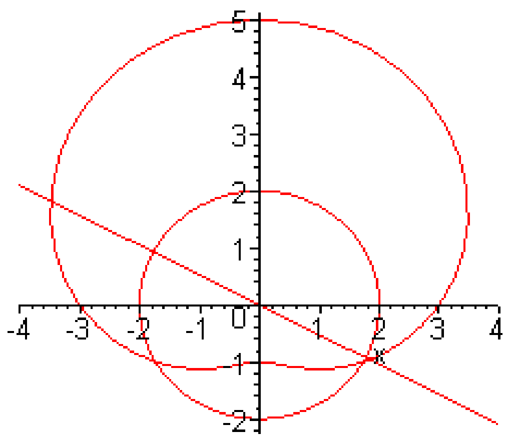
Problem # 33

```
> r1(t):=3+2*sin(t); r2(t):=2;
```

$$\mathrm{r1}(t) := 3 + 2\sin(t)$$

$$\mathrm{r2}(t) := 2$$

```
> P1:=plot([r1(t),t,t=0..2*Pi],coords=polar):
P2:=plot([r2(t),t,t=0..2*Pi],coords=polar):
P3:=plot(-Pi/6*x,x=-4..4):
display(P1,P2,P3);
```




```
> solve(3+2*sin(t)=2,t);
```

$$-\frac{1}{6}\pi$$

$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \mathrm{r1}(t)^2\,dt + \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \mathrm{r2}(t)^2\,dt$$

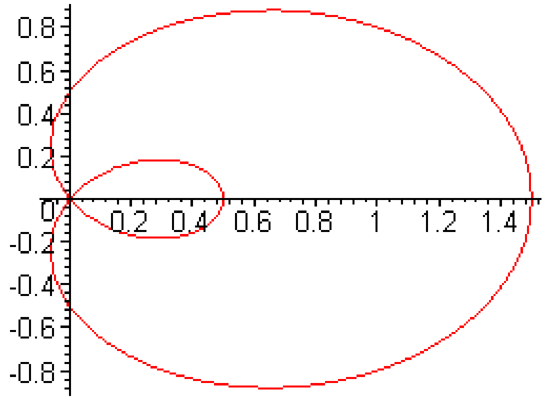
```
> 2*(1/2*int(r1(t)^2,t=-Pi/2..-Pi/6)+1/2*int(r2(t)^2,t=-
Pi/6..Pi/2));
```

$$-\frac{11}{2}\sqrt{3}+\frac{19}{3}\pi$$

```
> r(t):=1/2+cos(t);
```

$$r(t) := \frac{1}{2} + \cos(t)$$

```
> plot([r(t),t,t=0..2*Pi],coords=polar);
```



Evaluate the integral

$$\int_0^{\frac{2\pi}{3}} r(t)^2\, dt - \int_{\frac{2\pi}{3}}^{\pi} r(t)^2\, dt$$

```
> 2*(1/2*int(r(t)^2,t=0..2*Pi/3)-1/2*int(r(t)^2,t=2*Pi/3..Pi));
```

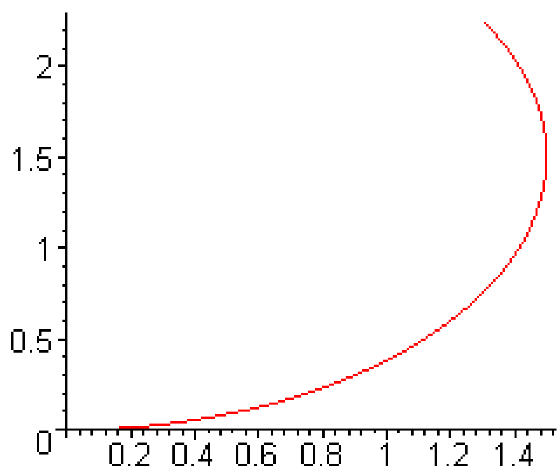
$$\frac{3}{4}\sqrt{3} + \frac{1}{4}\pi$$

Problem # 45

```
> r(t):=3*sin(t);
```

$$r(t) := 3\sin(t)$$

```
> plot([r(t),t,t=0..Pi/3], coords=polar);
```



To find the arc length, we evaluate the integral $\int_0^{\frac{\pi}{3}} \sqrt{r(t)^2 + \left(\frac{\partial}{\partial t} r(t)\right)^2}\, dt$

```
> int(sqrt(r(t)^2+diff(r(t),t)^2),t=0..Pi/3);
```

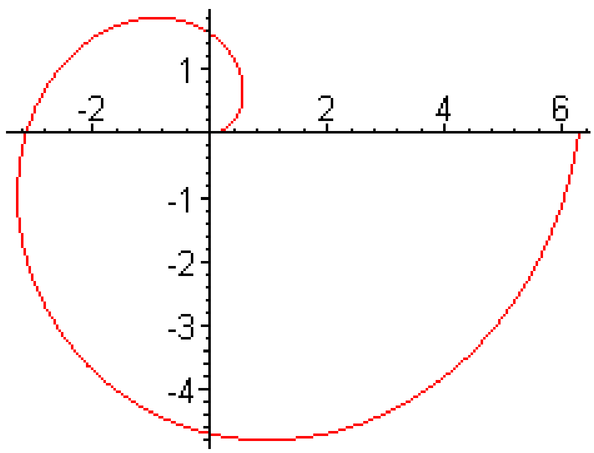
$$\pi$$

Problem#48

```
> r(t):=t;
```

$$r(t) := t$$

```
> plot([r(t),t,t=0..2*Pi],coords=polar);
```



We evaluate the integral $\int_0^{2\pi} \sqrt{r(t)^2 + \left(\frac{\partial}{\partial t} r(t)\right)^2}\, dt$

```
> int(sqrt(r(t)^2+diff(r(t),t)^2),t=0..2*Pi);
```

$$\sqrt{4\pi^2+1}\,\pi - \frac{1}{2}\ln\left(-2\pi + \sqrt{4\pi^2+1}\right)$$

\>