Math 201
Maple Handout # 10.2
Calculus with Parametric Curves

NOTE: To type click on T icon. To insert > for typing math, click on [> icon
Whenever you open a Maple file, press ENTER with cursor anywhere on **restart:** and on **with(plots):**

```
> restart:
```

This command clears maple memory and assign new values to variable.

```
> with(plots):
```

This command includes maple inbuilt package "plots"

```
Warning, the name changecoords has been redefined
```

Define a parametric curve

```
> f(t):=r*(t-sin(t)); g(t):=r*(1-cos(t));
```

$$f(t) := r\,(t - \sin(t))$$

$$g(t) := r\,(1 - \cos(t))$$

```
> diff(f(t),t);
```

$$r\,(1 - \cos(t))$$

```
> diff(g(t),t);
```

$$r\sin(t)$$

Area under a Parametric Curve $\int_0^{2\pi} g(t)\left(\frac{\partial}{\partial t} f(t)\right) dt$

```
> Area:=int(g(t)*diff(f(t),t),t=0..2*Pi);
```

$$Area := 3\pi r^2$$

Arc Length $\int_0^{2\pi} \sqrt{\left(\frac{\partial}{\partial t} f(t)\right)^2 + \left(\frac{\partial}{\partial t} g(t)\right)^2}\, dt$

```
> Arc:=int(sqrt(diff(f(t),t)^2+diff(g(t),t)^2),t=0..2*Pi);
```

$$Arc := 8\frac{r^2}{\sqrt{r^2}}$$

Surface Area of revolution about x-axis

$$2\pi \int_0^{\pi} g(t)\sqrt{\left(\frac{\partial}{\partial t} f(t)\right)^2 + \left(\frac{\partial}{\partial t} g(t)\right)^2}\, dt$$

```
> f(t):=r*cos(t); g(t):=r*sin(t);
```

$$f(t) := r\cos(t)$$

$$g(t) := r\sin(t)$$

```
> SA:=2*Pi*int(g(t)*sqrt(diff(f(t),t)^2+diff(g(t),t)^2),t=0..Pi);
```

$$SA := 4\pi\sqrt{r^2}\, r$$

Exercise 10.2
Prob#6

```
> f(t):=cos(t)+sin(2*t);
```

$$f(t) := \cos(t) + \sin(2t)$$

```
> g(t):=sin(t)+cos(2*t);
```

$$g(t) := \sin(t) + \cos(2t)$$

```
> animatecurve([f(t),g(t),t=0..2*Pi],frames=200,numpoints=100);
```

Derivative $\frac{dy}{dx} = \frac{\frac{\partial}{\partial t} g(t)}{\frac{\partial}{\partial t} f(t)}$

```
> dd:=diff(g(t),t)/diff(f(t),t);
```

$$dd := \frac{\cos(t) - 2\sin(2t)}{-\sin(t) + 2\cos(2t)}$$

Substitute t = 0 in the derivative

```
> m:=evalf(subs(t=0,dd));
```

$$m := .50000000($$

Substitute t = 0 in *f(t)* and *g(t)*

```
> x1:=evalf(subs(t=0,f(t)));
```

$$x1 := 1.$$

```
> y1:=evalf(subs(t=0,g(t)));
```

$$y1 := 1.$$

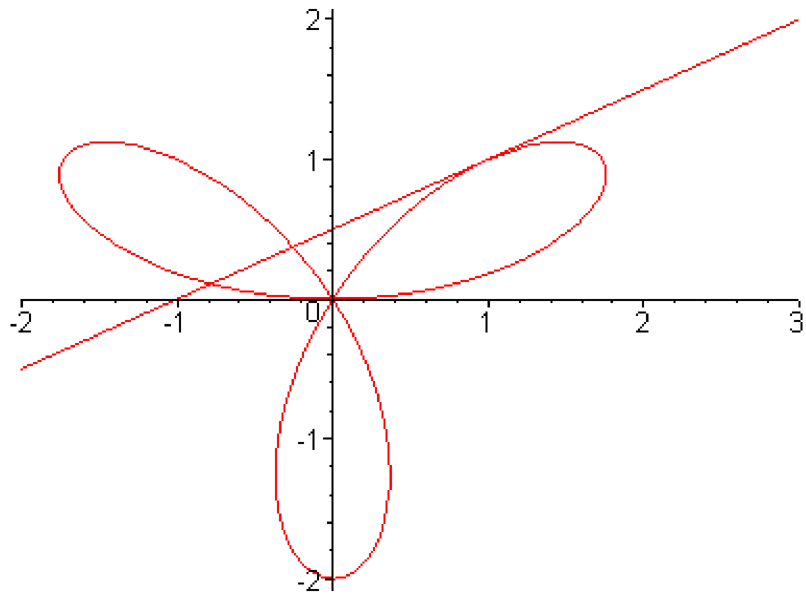
Equation of the tangent line: *(y - y1) = m (x - x1)*

```
> y:=x->m*(x-x1)+y1;
>
```

$$y := x \to m(x - x1) + y1$$

Compute two plots and display in the same window

```
> plot1:=plot([f(t),g(t),t=0..2*Pi]):
> plot2:=plot(y(x),x=-2..3):
> display(plot1,plot2);
```



>

Prob#9

```
> x1:=t->2*sin(2*t); y1:=t->2*sin(t);
```

$$x1 := t \to 2\sin(2t)$$

$$y1 := t \to 2\sin(t)$$

```
> diff(x1(t),t);
```

$$4\cos(2t)$$

```
> diff(y1(t),t);
>
```

$$2\cos(t)$$

```
> solve(x1(t)=sqrt(3),t);
```

$$\frac{1}{6}\pi$$

```
> solve(y1(t)=1,t);
```

$$\frac{1}{6}\pi$$

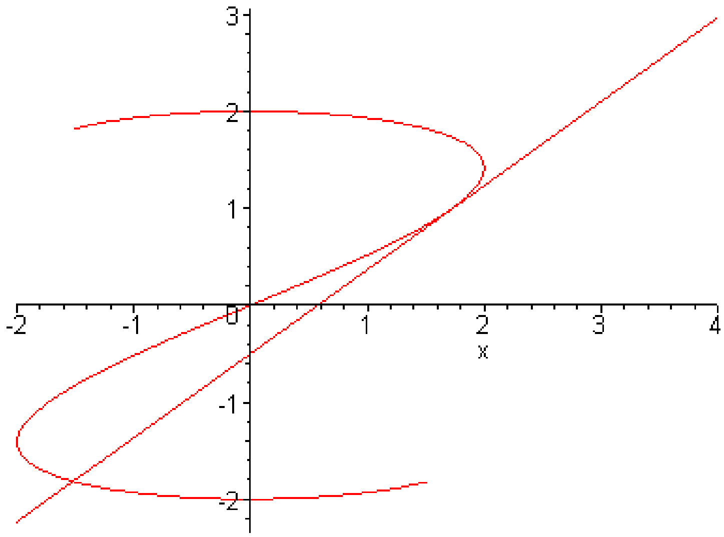
```
> m:=subs(t=Pi/6,(diff(y1(t),t)/diff(x1(t),t)));
```

$$m := \frac{1}{2}\frac{\cos\left(\frac{1}{6}\pi\right)}{\cos\left(\frac{1}{3}\pi\right)}$$

```
> y:=x->m*(x-sqrt(3))+1;
```

$$y := x \to m\left(x - \sqrt{3}\right) + 1$$

```
> Plt1:=plot(y(x),x=-2..4):
> Plt2:=plot([x1(t),y1(t),t=-2..2]):
display(Plt1,Plt2);
```




```
>
```