Solution Math 131 Quiz 4 (A)

Q.1: An initial investment of \$2600 grows at an annual rate of 7.5% compounded monthly. Find how long it takes for the investment to amount to \$3500.

Sol: Here $P = 2600,\ S = 3500,\ r = 7.5\% = 0.075,\ n = 12$. We need to find time (number of years) t.

The formula $S = P\left(1+\frac{r}{12}\right)^{nt}$ gives $3500 = 2600\left(1+\frac{0.075}{12}\right)^{12t}$. Dividing 3500 by 2600 and taking log on both sides.gives $12t\ \ln\left(1+\frac{0.075}{12}\right) = \ln\left(\frac{35}{26}\right)$ or $t = \frac{\log\left(\frac{35}{26}\right)}{12\log\left(1+\frac{0.075}{12}\right)} = 3.975 = 4$.

Q.2: A trust fund for a child's education is being set up by a single payment so that at the end of 17 years there will be \$31,000. If the fund earns interest at the rate of 8.25% compounded monthly, how much money should be paid into the fund initially?

Sol: Here $S = 31,000,\ r = 8.25\% = 0.0825,\ n = 12,\ t = 17$ years. We need to find the initial amount P.

The formula $P = S\left(1+\frac{r}{12}\right)^{-nt}$ gives $P = 31000\left(1+\frac{0.0825}{12}\right)^{-12(17)} = 7662.1$.

Q.3: If a person deposits \$1000 in a savings account that pays an interest rate of r% compounded continuously, and the account has \$1400 at the end of 4 years, find the interest rate.

Sol: Here $P = 1000,\ S = 1400,\ t = 4\ years$ We need to find time the interest rate r.

The formula $S = Pe^{rt}$ gives $1400 = 1000e^{4r}$. Dividing 1400 by 1000 and taking natural logarithm on both sides.gives $r = \frac{\ln(1.4)}{4} = 0.0841\,8 = 8.418\%$.

Q.4: Suppose a person deposits \$1000 in a savings account at the end of every six months. What is the value of the account at the end of five years if interest is at a rate of 10% compounded semiannually?

Sol: Here $R = 1000,\ n = 2$ (number of payment periods in one year), $t = 5\ years,\ r = \frac{10\%}{2} = 0.05$ (semiannually).

The formula $A = R\left[\frac{(1+r)^{nt}-1}{r}\right] = 1000\left(\frac{(1+0.05)^{10}-1}{0.05}\right) = 12578..$