Math 131 Quiz 3 (A)

Name:...Serial #:...

Q.1: Use simplex method to maximize $Z = 8x_1 + 2x_2$ subject to the constraints

$$\begin{aligned} x_1 - x_2 &\le 1 \\ x_1 + 2x_2 &\le 8 \\ x_1 + x_2 &\le 5 \\ x_1, x_2 &\ge 0 \end{aligned}.$$

Sol: Let s_1, s_2, and s_3 be nonnegative real numbers such that $\begin{aligned} x_1 - x_2 + s_1 &= 1 \\ x_1 + 2x_2 + s_2 &= 8 \\ x_1 + x_2 + s_3 &= 5 \end{aligned}$ and write the objective function as $-8x_1 - 2x_2 + Z = 0$. Then the first Simplex method table is:

B	x_1	x_2	s_1	s_2	s_3	Z	R
s_1	1	-1	1	0	0	0	1
s_2	1	2	0	1	0	0	8
s_3	1	1	0	0	1	0	5
Z	-8	-2	0	0	0	1	0

The most negative entry in the last row is -8 and smallest positive ratio of the entries in the last column by the entries in the x_1 column is 1 which makes the entry in the first row and x_1 column as pivot entry. Also x_1 si the entring variable and s_1 is the departing variable. Now we will use pivot entry to make all other entries in the x_1 column equal to 0 using the row poeration: $-R_1 + R_2,\ -R_1 + R_3,$ and $8R_1 + R_4$.

B	x_1	x_2	s_1	s_2	s_3	Z	R
x_1	1	-1	1	0	0	0	1
s_2	0	3	-1	1	0	0	7
s_3	0	2	-1	0	1	0	4
Z	0	-10	8	0	0	1	8

. The most negative entry in the last row is -10 and smallest positive ratio of the entries in the last column by the entries in the x_2 column is 2 which makes the entry in the third row and x_2 column as pivot entry. Since the pivot entry is not 1, we divide the third row by 2 to make it equal to 1.

B	x_1	x_2	s_1	s_2	s_3	Z	R
x_1	1	-1	1	0	0	0	1
s_2	0	3	-1	1	0	0	7
x_2	0	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	2
Z	0	-10	8	0	0	1	8

Now we will use pivot entry to make all other entries in the x_1 column equal to 0 using the row poeration: $R_3 + R_1,\ -3R_3 + R_2$, and $10R_3 + R_4$.

B	x_1	x_2	s_1	s_2	s_3	Z	R
x_1	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	3
s_2	0	0	$\frac{1}{2}$	1	$-\frac{3}{2}$	0	1
x_2	0	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	2
Z	0	0	3	0	5	1	28

. Since all the entries in the last row are nonnegative, we stop the method and find that the maximum value of $Z = 28$ for $x_1 = 3$ and $x_2 = 2$.

Q.2: Find Dual and use simplex method to solve the problem. Minimize $Z = x_1 + x_2 + 2x_3$ subject to the constraints

$$\begin{aligned} x_1 - x_2 - x_3 &\le 1 \\ -x_1 - x_2 + x_3 &\ge 3 \\ x_1, x_2, x_3 &\ge 0 \end{aligned} .$$

Sol: Rewrite the first inequality as: $$\begin{aligned} -x_1 + x_2 + x_3 &\ge -1 \\ -x_1 - x_2 + x_3 &\ge 3 \\ x_1, x_2, x_3 &\ge 0 \end{aligned} .$$

The dual maximization problem is as follows:

Maximize $W = -y_1 + 3y_2$

Subject to the constraints $$\begin{aligned} -y_1 - y_2 &\le 1 \\ y_1 - y_2 &\le 1 \\ y_1 + y_2 &\le 2 \\ y_1,\ y_2,\ y_3 &\ge 0 \end{aligned} .$$