Solution Math 131 - 02, Summer 073
Midterm Exam # 2, August 13, 2008

Q.1: An investment grows from \$600 to \$642 in one year at an interest rate r such that interest is compounded continuously. Find the interest rate r and use it to find the number of years it will take for the investment to double. (10 pts)

Sol: We shall use $S = pe^{rt}$ with $S = 642,\ P = 600$, and $t = 1$ to find r.

$642 = 600e^r$ gives $r = \ln\left(\dfrac{642}{600}\right) = 0.0676\,9$.

Now use this r and solve $2P = Pe^{0.0676\,9t}$

$t = \dfrac{\ln(2)}{0.06769} = 10.24$ years.

Q.2: You have a choice of two banks. One bank pays interest at 10% compounded monthly and the other bank pays interest at 9% compounded daily (365 times a year). Which is the better choice? How much more would you make in one year if you deposited \$1000? (10 pts)

Sol: $S_1 = P\left(1 + \dfrac{r}{12}\right)^{12} = 1000\left(1 + \dfrac{0.10}{12}\right)^{12} = 1104.7$

$S_2 = P\left(1 + \dfrac{r}{365}\right)^{365} = 1000\left(1 + \dfrac{0.09}{365}\right)^{365} = 1094.2$

Thus first choice is better.

Q.3 Suppose that you can invest \$5000 in a business that guarantees you the following cash flows: \$3000 at the end of 2 years, \$2000 at the end of 4 years, and \$2000 at the end of 6 years. Assuming an interest rate of 6% compounded monthly, find the present value of the cash flows. Is the investment profitable? (10 pts)

Sol: Initial value of these cash flows is

$$P = 3000\left(1 + \frac{0.06}{12}\right)^{-24} + 2000\left(1 + \frac{0.06}{12}\right)^{-48} + 2000\left(1 + \frac{0.06}{12}\right)^{-72} = 5632.4.$$

Since $P > 5000$, therefore the investment is profitable and the profit is \$132.4.

Q.4: A woman makes house payments of \$4200 at the beginning of every quarter. If the woman wishes to pay $1\frac{1}{2}$ year's worth of payments in advance, how much should she pay provided that the interest rate is 5.4% compounded quarterly? (8 pts)

Sol: We need to find present value of an anuity due with $R = 4200,\ r = 0.054,\ n = 6$ (6 qrauters in $1\frac{1}{2}$ years).

$$A = R(1+r)\left(\frac{1-(1+r)^n}{r}\right) = 4200\left(1 + \frac{0.054}{4}\right)\left(\frac{1-\left(1+\frac{0.054}{4}\right)^{-6}}{\frac{0.054}{4}}\right) = 24376.$$

Q.5: If \$5000 is used to purchase an annuity consisting of equal payments at the end of each month for the next $3\frac{1}{2}$ year's and the interest rate is 6% compounded monthly, find the amount of each payment. (8 pts)

Sol: We need to find R for a simple anuity with initial value \$5000, $r = 0.06,\ n = 42$ (42 months in $3\frac{1}{2}$ years)

$$R = A\left(\frac{(1+r)^n - 1}{r}\right)^{-1} = 5000\left(\frac{1-\left(1+\frac{0.06}{12}\right)^{-42}}{\frac{0.06}{12}}\right)^{-1} = 132.28.$$

Q.6: A club has ten members. In how many ways can the offices of president, vice president, secretary, and treasurer be filled if no member can serve in two offices? (Note: Order is important) (6pts)

Sol: $10.9.8.7 = 5040.$

Q.7: Five different books are to be arranged horizontally on a bookshelf. (a) In how many ways can this be done? (b) If two are mathematics books and three are accounting books, in how many ways can all the books be arranged if the first two books are to be in mathematics? (8 pts)

Sol: (a) $5! = 5.4.3.2.1 = 120$

(b) $3!2! = 12.$

Q.8: A student must select two courses in the liberal arts and three courses in the social sciences. There are six liberal arts courses and ten social science courses, all of which are different, from which the student may choose. How many selections are possible? (8 pts)

Sol: ${}_6C_2 \cdot {}_{10}C_3 = \frac{6!}{4!2!}\frac{10!}{7!3!} = 1800$

Q.9: A coin is tossed three times. Determine

(a) the event that exactly two heads occur;

(b) the event that at least two heads occur;

(c) the event that no head occurs. (6 pts)

Sol: $S = \{HHH,\ HHT,\ HTH,\ THH,\ HTT,\ THT,\ TTH,\ TTT\}$

(a) $E = \{HHT,\ HTH,\ THH\}$

(b) $E = \{HHT,\ HTH,\ THH,\ HHH\}$

(c) $E = \{TTT\}$

Q.10: An urn contains five red and three green marbles. Two marbles are randomly drawn in succession without replacement. Determine the probability that

(a) the first marble is red and the second is green;

(b) both marbles are red.

(c) one marble is red and other second is green. (9 pts)

Sol: $(a)\ P(E) = \frac{5}{8}\frac{3}{7} = \frac{15}{56}$,

$(b)\ P(E) = \frac{5}{8}\frac{4}{7} = \frac{5}{14}$,

$(c)\ P(E) = \frac{5}{8}\frac{3}{7} + \frac{3}{8}\frac{5}{7} = \frac{15}{28}$.

Q.11:If $P(E) = 0.3, P(F) = 0.4, P(E \cap F) = 0.2$, find $P(E \cup F)$. (5 pts)

Sol: $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.3 + 0.4 - 0.2 = 0.5$.

Q.12: Use the dual and the simplex method to

minimize

$Z = 4x_1 + 5x_2$

subject to

$x_1 - x_2 \geq 4,$

$2x_1 - x_2 \geq 1,$

$5x_1 + 3x_2 \geq 3,$

$x_1,\ x_2, x_3 \geq 0.$

Sol: The Dual problem is

Maximize $W = 4y_1 + y_2 + 3y_3$

Subject to the constraints

$y_1 + 2y_2 + 5y_3 \leq 4$

$-y_1 - y_2 + 3y_3 \leq 5$

$y_1,\ y_2,\ y_3 \geq 0$

Let s_1, and s_2 be nonnegative real numbers such that $y_1 + 2y_2 + 5y_3 + s_1 = 4$, and $-y_1 - y_2 + 3y_3 + s_2 = 5$. Also write $-4y_1 - y_2 - 3y_3 + W = 0$. Then the first method table is:

B	y_1	y_2	y_3	s_1	s_2	W	R
s_1	1	2	5	1	0	0	4
s_2	-1	-1	3	0	1	0	5
W	-4	-1	-3	0	0	1	0

. Pivot entry is 1 in the y_1 column. Use row operations $R_1 + R_2$ and $4R_1 + R_3$ to get the second table

B	y_1	y_2	y_3	s_1	s_2	W	R
y_1	1	2	5	1	0	0	4
s_2	0	1	8	1	1	0	9
W	0	7	17	4	0	1	16

. Thus the minumum value of $Z = 16$, for $x_1 = 4$ and $x_2 = 0$.