Math 131 - 03, Summer 073

Solution Midterm Exam # 1, July 23, 2008

SHORT ANSWER. Solve all the questions. Show all the steps to get full credit.

1) If the demand equation for a certain product is $p = 400 - q^2$ and its supply equation is $p = 20q + 100$. Find the equilibrium quantity and equilibrium price.

Sol: $-q^2+400 = 20q+100$

$q^2+20q-300 = 0$

$(q-10)(q+30) = 0$

$q = 10$, and so $p = 300$

2) A person wishes to deposit a total fo $10,000 in two accounts. The savings account pays yearly interest of 4% and fixed certificates of deposit pay a yearly interest rate of 7%. How much should the person deposit in each account so that he gets a total of $502 interest at the end of the year?

Sol: Let x = money deposited into saving account. Then

$0.04x+0.07(10000-x) = 502$

$-0.03x = 502-700$

$x = \dfrac{198}{0.03} = 6600$ is deposited into saving account and 3400 is deposited into certificated.

3) Suppose a company offers you a sales position with your choice of two methods of determining your yearly salary. One method pays $15,000 plus a bonus of 3% of your yearly sales. The other method pays a straight 13% commission of your sales. For what yearly sales level is it better to choose the first method?

Sol: Let x be the yearly sale, then

$0.03x + 15000 > 0.13x$

$x < \dfrac{15000}{0.10} = 150,000$

4) The stock price of a company has risen at the rate of $5.00 per month over the last year. On January 1 it was $75.00. Write an equation that shows this relationship.

Sol: The price equation is $y = 5x + c$, and on January 1, $x = 0$ and $y = 75$.

So $75 = 0 + c$ gives $c = 75$

Thus $y = 5x + 75$.

5) A company will manufacture a total of 5000 units of its product at plants A and B. At plan A the unit cost for labor and material combined is $2.50, while at plant B it is $3.00. The fixed costs at plant A are $6000 and at plant B they are $8000. Between the two plants the company has decided to allot no more than $28,000 for total costs. Find the minimum number of units that must be produced at plant A.

Sol: Let x be the number of units produced at plant A, then $5000-x$ units are produced at plant B.

So, $2.5x + 6000 + 3(5000-x) + 8000 \leq 28000$

$-0.5x \leq 28,000-29,000$

$x \geq 2000$.

6) The demand per week for a new automobile is 400 units when the price is \$16,700 each, and 500 units when the price is \$14,900 each. Find the demand equation for the cars, assuming that it is linear.

Sol: The two points are (400, 16700) and (500, 14900)

$$\text{The slope is } m = \frac{14900\text{-}16700}{500\text{-}400} = -\frac{1800}{100} = -18$$

p-14900 = -18(q - 500)
p = -18q + 23900

7) When the temperature T (in degrees Celsius) of a certain laboratory animal is reduced, its heart rate r (in beats per minute) decreases. At a temperature of 37°C, the animal had a heart rate of 200, and at a temperature of 32°C its heart rate was 140. If r is a linear function of T for $26 \leq T \leq 38$, (a) determine this function and (b) determine the heart rate at a temperature of 30°C.

Sol: The two points are (37, 200) and (32, 140)

(a) $$\text{The slope is } m = \frac{200\text{-}140}{37\text{-}32} = \frac{60}{5} = 12$$

r-200 = 12(T - 37)
r = 12T - 244

$$\text{OR } T = \frac{1}{12}r + \frac{244}{12}$$

(b) at T = 30, r = 360 - 244 = 116.

8) Suppose that the vertex of the parabola $y = 3x^2 - 6x + k$ is (1, 2); find k.

Sol: Vertex is, x = 1 and y = 2,
But at $x = 1$, $y = 3 - 6 + k = 2$ gives $k = 5$.

9) A young family with two children has \$40,000 saved for college costs, with part invested at 12% and part invested at 8%. If the total yearly income from the investments is \$3400, how much is invested at each rate?

Sol: Let x be invested at 12% and y be invested at 8%. Then x + y = 40,000 and 0.12x + 0.08y = 3400.
0.12(40,000 - y) + 0.08y = 3400
- 0.04y=3400 - 4800

$$y = \frac{1400}{0.04} = 35{,}000 \text{ is invested at 8\% and 5000 is invested at 12\%.}$$

10) Find all the solutions of the system $\begin{cases} x^2 + y^2 = 7 \\ x^2 - y^2 = 1 \end{cases}$

Sol: 2 x^2 = 8 or x^2 = 4 or x = ±2
and y^2 = 7 - 4 =3 gives y = $\pm\sqrt{3}$
So all the solutions are $(\pm 2, \pm\sqrt{3})$

11) For the parabola $y = f(x) = 2 - 3x - 5x^2$, find: (a) the vertex, (b) the y-intercept, and (c) the x-intercepts.

Sol: Here a = -5, b = -3, and c = 2.

The vertex is at $x = -\frac{b}{2a} = -\frac{-3}{-10} = -\frac{3}{10}$ and $y = -5(-\frac{3}{10})^2 - 3(-\frac{3}{10}) + 2 = \frac{245}{100}$

For y-intercept, put $x = 0$, then y = 2,

For x-intercepts, put $y = 0$, then $5x^2 + 3x - 2 = 0$, gives $x = \frac{-3 \pm \sqrt{9 + 40}}{10} = -1, 0.4$

12) A manufacturer sells his product at \$12.50 per unit, selling all the produces. His fixed cost is \$5,000 and his variable cost per unit is \$8.50. (a) At what level of production will he break even? (b) At what level of production will he have a profit of \$10,000?

Let x be the number of units. Then $8.5x + 5000 = 12.5x$ gives $x = 1250$ units.
The profit is 10,000 if $12.5x - 8.5x - 5000 = 10{,}000$ or $x = 3{,}750$ units are sold.

13)

Using the method of reduction, solve the system: $\begin{cases} 3x - 2y + z = -1 \\ 2x - y - z = 5 \\ 2x + 3z = 4 \end{cases}$

Sol: $\begin{bmatrix} 3 & -2 & 1 & -1 \\ 2 & -1 & -1 & 5 \\ 2 & 0 & 3 & 4 \end{bmatrix} \xrightarrow{-R_2+R_1} \begin{bmatrix} 1 & -1 & 2 & -6 \\ 2 & -1 & -1 & 5 \\ 2 & 0 & 3 & 4 \end{bmatrix} \xrightarrow[-2R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & -1 & 2 & -6 \\ 0 & 1 & -5 & 17 \\ 0 & 2 & -1 & 16 \end{bmatrix} \xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & -1 & 2 & -6 \\ 0 & 1 & -5 & 17 \\ 0 & 0 & 9 & -18 \end{bmatrix}$

$\xrightarrow[\frac{1}{9}R_3]{R_2+R_1} \begin{bmatrix} 1 & 0 & -3 & 11 \\ 0 & 1 & -5 & 17 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow[3R_3+R_1]{5R_3+R_2} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

This gives $x = 5$, $y = 7$, and $z = -2$.

14) Solve the following system of equations by reduction.

$\begin{cases} 2x + 3y + 4z + 9u = 4 \\ 3x + 4y + 8z + 7u = 1 \end{cases}$

Sol: $\begin{bmatrix} 2 & 3 & 4 & 9 & 4 \\ 3 & 4 & 8 & 7 & 1 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 2 & 3 & 4 & 9 & 4 \\ 1 & 1 & 4 & -2 & -3 \end{bmatrix} R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 1 & 4 & -2 & -3 \\ 2 & 3 & 4 & 9 & 4 \end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & 1 & 4 & -2 & -3 \\ 0 & 1 & -4 & 13 & 10 \end{bmatrix}$

$\xrightarrow{-R_2+R_1} \begin{bmatrix} 1 & 0 & 8 & -15 & -13 \\ 0 & 1 & -4 & 13 & 10 \end{bmatrix}$

$x = -8z + 15u - 13$, $y = 4z - 13u + 10$, $z = r$, $u = s$
where r and s are arbitrary.

15) A company has two different locations to assemble three different models of PCs. The table below summarizes the daily production capacity, the minimum number of each type needed, and the daily operating costs for each location. Find the number of days that each location needs to operate in order to fill the orders at minimum cost.

	Location 1	Location 2	Minimum Number
Model 1	60/day	60/day	2400
Model 2	40/day	80/day	2000
Model 3	60/day	40/day	1800
Weekly Cost	\$16,000	\$12,000	

Sol: Let x = number of days location1 need to operate

and y = number of days location1 need to operate, then

$60x + 60y \geq 2400$ OR $x + y \geq 40$,

$40x + 80y \geq 2000$ OR $x + 2y \geq 50$,

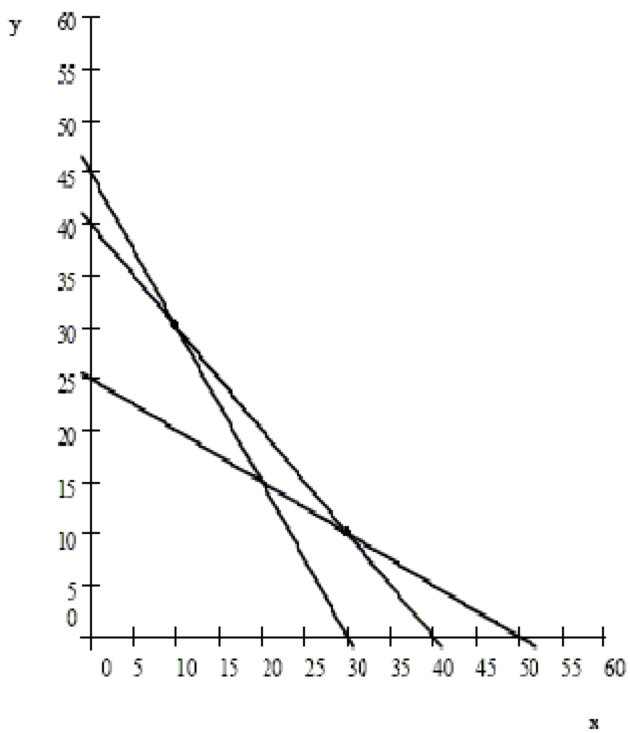
$60x + 40y \geq 1800$ OR $3x + 2y \geq 90$,

$x \geq 0$, $y \geq 0$,

Considering 5 working days in a week, the daily cost for location 1 is \$3200 and for location 2 it is \$2400.

The cost function to minimize is $Z = 3200x + 2400y$

The feasible region is



The four corner points are A(0,45), B(10,30), C(30,10), and D(50,0).

Z(A) = 108,000, Z(B) = 104,000, Z(C) = 120,000, and Z(D) = 160,000.

Thus cost is minimum at B(10,30) and minimum cost is \$104,000.