Math102
Chapter 5
Sec#5.1

```
>
> restart:
> with(student):
> with(plots):
Warning, the name changecoords has been redefined
```

Problem: Use leftpoint, middlepoint and rightpoint rule to approximate area under the graph of $f(x) = x^2$ from $x = 0$ to $x = 1$.

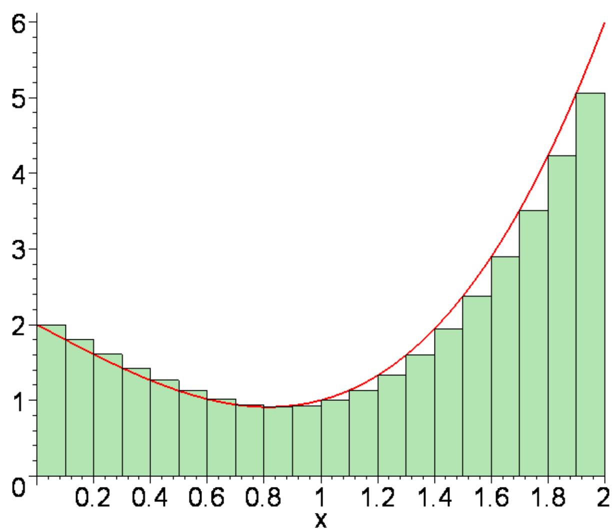
```
> f(x):=x^3-2*x+2;
```

$$f(x) := x^3 - 2\,x + 2$$

```
> n:=20;
```

$$n := 20$$

```
> leftbox(f(x), x=0..2,n);
```



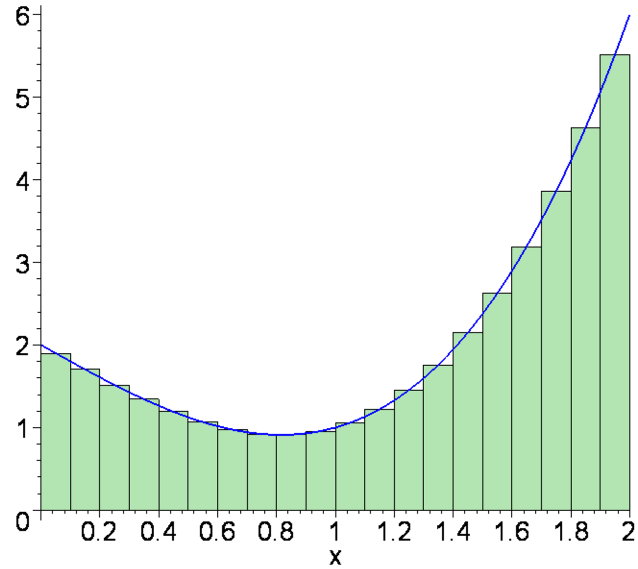

```
> leftsum(f(x), x=0..2,n);
```

$$\frac{1}{10}\left(\sum_{i=0}^{19}\left(\frac{1}{1000}\,i^3 - \frac{1}{5}\,i + 2\right)\right)$$

```
> evalf(%);
```

$$3.810000000$$

```
> middlebox(f(x), x=0..2,n, color=BLUE);
```



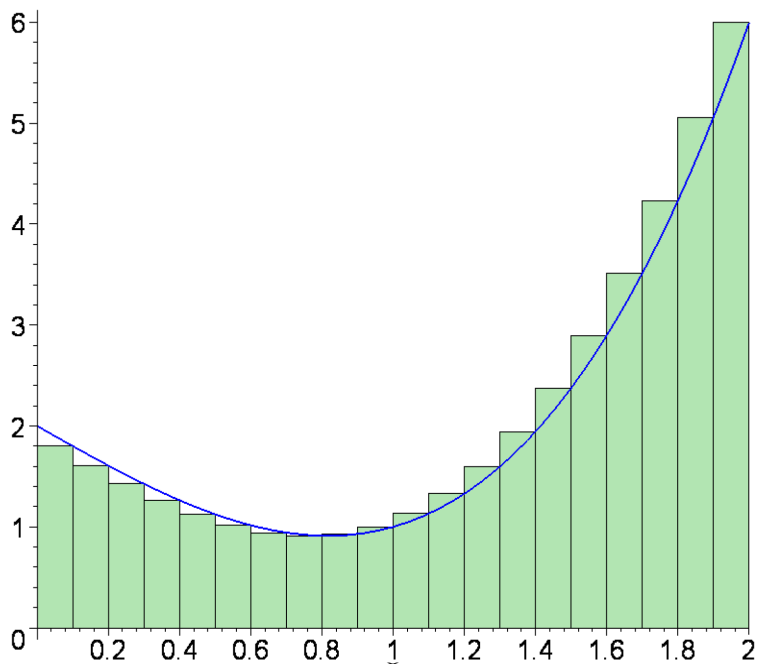

```
> middlesum(f(x), x=0..2,n, color=BLUE);
```

$$\frac{1}{10}\left(\sum_{i=0}^{19}\left(\left(\frac{1}{10}i+\frac{1}{20}\right)^3-\frac{1}{5}i+\frac{19}{10}\right)\right)$$

```
> evalf(%);
```

$$3.995000000$$

```
> rightbox(f(x), x=0..2,n, color=BLUE);
```




```
> rightsum(f(x), x=0..2,n, color=BLUE);
```

$$\frac{1}{10}\left(\sum_{i=1}^{20}\left(\frac{1}{1000}i^3-\frac{1}{5}i+2\right)\right)$$

```
> evalf(%);
```

$$4.210000000$$

Exact Integral

```
> Int(f(x),x=0..2)=evalf(int(f(x),x=0..2));
```

$$\int_0^2 x^3 - 2\,x + 2\,dx = 4.$$

with $n = 10$.

```
> g(x):=5*x*(x-1)^2; n:=10;
```

Problem: Use leftpoint, middlepoint and rightpoint rule to approximate area under the graph of $f(x) = 5\,x\,(x-1)^2$ from $x = 0$ to $x = 1$.

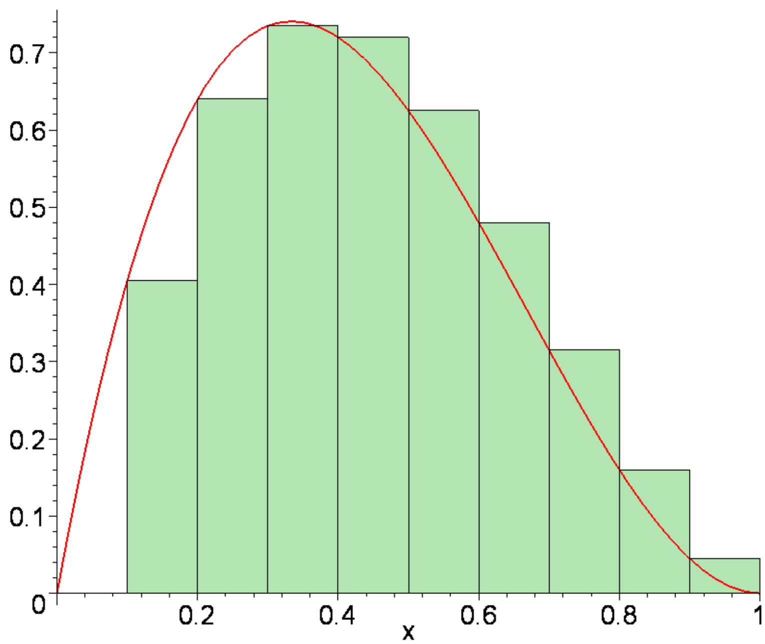
$$g(x) := 5\,x\,(x-1)^2$$

$$n := 10$$

```
> g(x) := 5*x*(x-1)^2;
```

$$g(x) := 5\,x\,(x-1)^2$$

```
> leftbox(g(x), x=0..1,n);
```



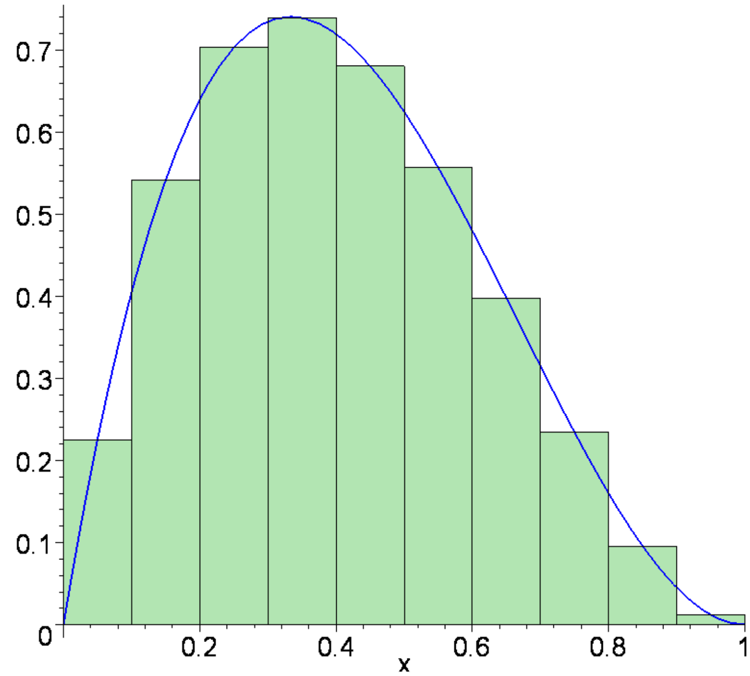

```
> leftsum(g(x), x=0..1,n);
```

$$\frac{1}{10}\left(\sum_{i=0}^{9}\left(\frac{1}{2}\,i\left(\frac{1}{10}\,i-1\right)^2\right)\right)$$

```
> evalf(%);
```

$$.4125000000$$

```
> middlebox(g(x), x=0..1,n, color=BLUE);
```



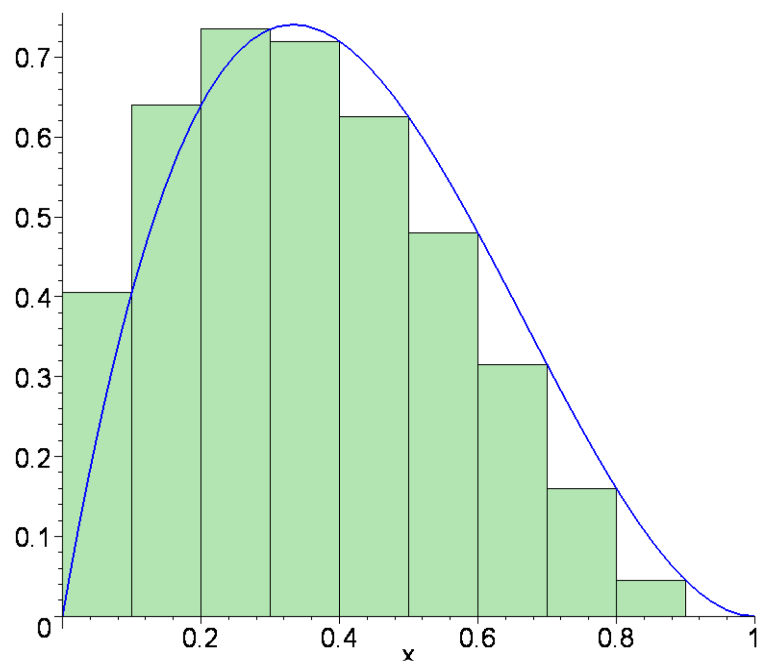

```
> middlesum(g(x), x=0..1,n, color=BLUE);
```

$$\frac{1}{10}\left(\sum_{i=0}^{9}\left(5\left(\frac{1}{10}i+\frac{1}{20}\right)\left(\frac{1}{10}i-\frac{19}{20}\right)^2\right)\right)$$

```
> evalf(%);
```

$$.4187500000$$

```
> rightbox(g(x), x=0..1,n, color=BLUE);
```




```
> rightsum(g(x), x=0..1,n, color=BLUE);
```

$$\frac{1}{10}\left(\sum_{i=1}^{10}\left(\frac{1}{2}\,i\left(\frac{1}{10}\,i-1\right)^2\right)\right)$$

```
> evalf(%);
```

$$.4125000000$$

Exact Value

```
> Int(g(x),x=0..1)=evalf(int(g(x),x=0..1));
>
```

$$\int_0^1 5\,x\,(x-1)^2\,dx = .4166666667$$

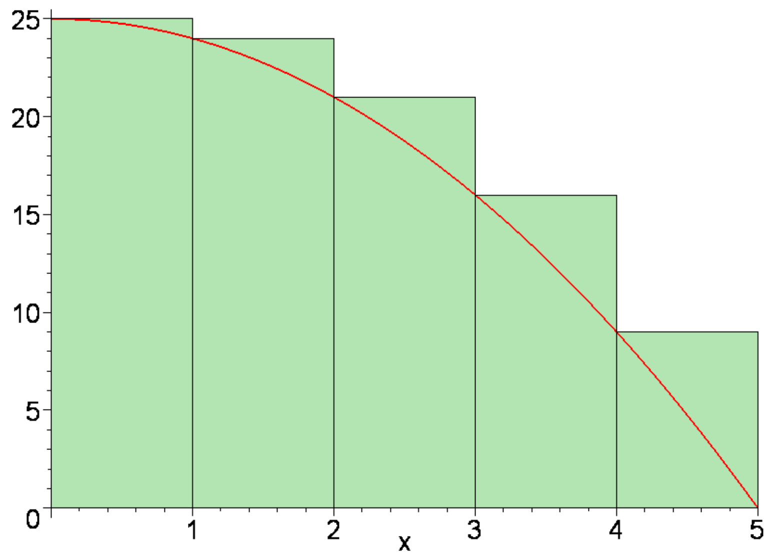
Problem: Use leftpoint, middlepoint and rightpoint rule to approximate area under the graph of $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$.

```
> h(x):=25-x^2; n:=5;
```

$$h(x) := 25 - x^2$$

$$n := 5$$

```
> leftbox(h(x),x=0..5,n);
```



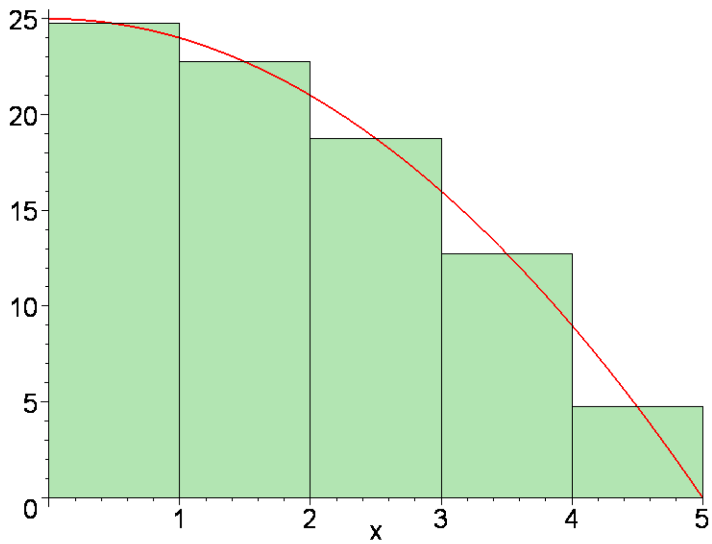

```
> leftsum(h(x),x=0..5,n);
```

$$\sum_{i=0}^{4}(25 - i^2)$$

```
> evalf(%);
```

$$95.$$

```
> middlebox(h(x),x=0..5,n);
```



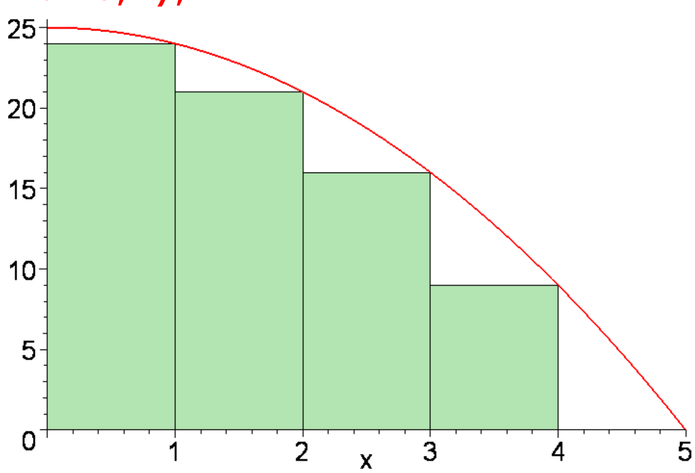

```
> middlesum(h(x),x=0..5,n);
```

$$\sum_{i=0}^{4}\left(25-\left(i+\frac{1}{2}\right)^2\right)$$

```
> evalf(%);
```

$$83.75000000$$

```
> rightbox(h(x),x=0..5,n);
```




```
> rightsum(h(x),x=0..5,n);
```

$$\sum_{i=1}^{5}(25-i^2)$$

```
> evalf(%);
```

$$70.$$

Exact Value

```
> Int(h(x),x=0..5)=evalf(int(h(x),x=0..5));
```

$$\int_0^5 25-x^2\,dx=83.33333333$$

NOTE: If a function is increasing, then use of left end points gives an underestimate and use of right end points gives an overestimate. Similarly if a function is decreasing, then use of left end points gives an overestimate and use of right end points gives an underestimate.

>

>