Formula Sheet for Math 301-102 Final Exam

1. $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = F(s)$ 2. $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

3. $\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$

4. $\mathcal{L}\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n}[F(s)]$ 5. $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$

6. $\mathcal{L}\{f(t)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$ 7. $\mathcal{L}(\delta(t-t_0)) = e^{-st_0}$

8. $f \star g = \int_0^t f(\tau)g(t-\tau)d\tau$ 9. $\mathcal{L}(f \star g) = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\} = F(s)G(s)$

10. The Fourier Bessel series of f defined on the interval $(0,b)$ is $f(x) = \sum_{i=1}^\infty c_i J_n(\alpha_i x)$, where $c_i = \dfrac{2}{b^2 J_{n+1}^2(\alpha_i b)} \int_0^b x J_n(\alpha_i x) f(x)dx$ and α_i are defined by $J_n(\alpha b) = 0$.

11. The Fourier Bessel series of f defined on the interval $(0,b)$ is $f(x) = \sum_{i=1}^\infty c_i J_n(\alpha_i x)$, where $c_i = \dfrac{2\alpha_i^2}{(\alpha_i^2 b^2 - n^2 + h^2)J_n^2(\alpha_i b)} \int_0^b x J_n(\alpha_i x) f(x)dx$ and α_i are defined by $hJ_n(\alpha b) + \alpha b J_n'(\alpha b) = 0$.

12. The Fourier Bessel series of f defined on the interval $(0,b)$ is $f(x) = c_1 + \sum_{i=2}^\infty c_i J_0(\alpha_i x)$, where $c_1 = \dfrac{2}{b^2}\int_0^b x f(x)dx$, $c_i = \dfrac{2}{b^2 J_0^2(\alpha_i b)} \int_0^b x J_0(\alpha_i x) f(x)dx$ and α_i are defined by $J_0'(\alpha b) = 0$.

13. The Fourier-Legendre series of f defined on the interval $(-1,1)$ is $f(x) = \sum_{n=0}^\infty c_n P_n(x)$, where $c_n = \dfrac{2n+1}{2}\int_{-1}^1 f(x)P_n(x)dx$.

14. Fourier transform: $\mathcal{F}\{f(x)\} = \int_{-\infty}^\infty f(x)e^{i\alpha x}dx = F(\alpha)$

15. Inverse Fourier tyransform: $\mathcal{F}^{-1}\{F(\alpha)\} = \frac{1}{2\pi}\int_{-\infty}^\infty F(\alpha)e^{-i\alpha x}d\alpha = f(x)$

16. Fourier sine transform: $\mathcal{F}_s\{f(x)\} = \int_0^\infty f(x)\sin(\alpha x)dx = F(\alpha)$

17. Inverse Fourier sine transform: $\mathcal{F}_s^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\sin(\alpha x)d\alpha = f(x)$

16. Fourier cosine transform: $\mathcal{F}_c\{f(x)\} = \int_0^\infty f(x)\cos(\alpha x)dx = F(\alpha)$

17. Inverse Fourier cosine transform: $\mathcal{F}_c^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\cos(\alpha x)d\alpha = f(x)$

18. Two recurrence relations

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x), \qquad \frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$$