King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

Final Exam for Math 302 – 03

Name:... .....Serial #:..........................

Note: Show all your work to get full credit.

Q.1: Find the general solution and write it as a column vector for the system

$$\begin{aligned} 7x - 3y + 4z &= -7 \\ 2x + y - z + 4w &= 6 \\ y - 3w &= -5 \end{aligned}$$

Q.2: Use Green's theorem to evaluate the integral $\oint_C (x^5 + 3y)\, dx + \left(2x - e^{y^3}\right) dy$, where C is the circle of radius 4 center at $(1, 5)$.

Q.3: Let $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$. Find eigenvalues and eigenvector. Check that the eigenvectors associated with distinct eigenvalues are orthogonal. Also write an orthogonal matrix that diagonalizes A.

Q.4: (a) Fine the curl and divergence of the function $F\,(x, y, z) = xye^{x}\ i - x^{3}yze^{z}\ j + xy^{2}e^{y}k$.
(b) Show that $div\,(F \times G) = G.curl\,(F) - F.curl\,(G)$, for any vector function F and G.

Q.5: Evaluate $\oint\limits_{C} \mathbf{F} \cdot d\mathbf{R}$, where $\mathbf{F} = [2\cos(2x)e^{y}]\, i + [e^{y}\sin(2x) - y]\ j$, and C is any path from $\left(\frac{\pi}{4}, 1\right)$ to $\left(\frac{3\pi}{4}, 2\right)$. (Hint: find potential function φ for $\mathbf{F}$ if it is conservative)

Q.6: Use Gauss's Divergence theorm to evaluate $\iint\limits_{\Sigma} F \cdot N d\sigma$, where $F = x^3\, i + y^3\, j + k$, Σ is the region bounded by $z = 2 - x^2 - y^2$,and $z = 0$. (Hint: perform integration with respect to $z, r,$ and θ).

Q.7: Use Stoke's theoren to evaluate $\iint\limits_{\Sigma} (\nabla \times \mathbf{F}) \cdot \hat{n}\, d\sigma$ or $\oint\limits_{C} F \cdot dR$ for $F = -y\ i{+}x\ j - xyz\ k$, and Σ is the part of the cone $z = \sqrt{x^2 + y^2}$, for $x^2 + y^2 \leq 9$.

Q.8: Determine the radius of convergence and open disk of convergence of nthe power series $\sum\limits_{n=0}^{\infty} \left(\frac{n^3}{4^n}\right) (z + 1 - 2i)^{3n}$

Q.9: Determine the fourth root of the complex number $-4i$, i.e. $(-4i)^{\frac{1}{4}}$.

Q.10: Evaluate $\int_{\Gamma} f(z)\, dz$, where $f(z) = |z|^2$, and Γ is the line segment from 0 to $1+i$.

Q.11: Evaluate $\oint\limits_{\Gamma} f(z)dz$, where $f(z) = z^2 \sin(z)$, where Γ is the square having vertices $0, 1, 1+i$, and i..

Q.12: Use Cauchy Integral formula to evaluate $\int_{\Gamma} f(z)dz$, where $f(z) = \dfrac{3z^2 \cosh(z)}{(z+2i)^2}$, Γ is a circle $|z-1| = 8$.

Q.13: Determine the Laurent series of the function $\dfrac{z - \sin(2z)}{z^3}$ and write all the singularities.

Q.14: Use Residue theorem to evaluate $\int\limits_{\Gamma} \dfrac{z - \cos(z)}{(1 + z^2)\, z^2} dz$, where Γ is any path that encloses i and 0 but not $-i$.

Q.15: Evaluate $\int\limits_{-\infty}^{\infty} \frac{\cos(\pi x)}{(4+x^2)(9+x^2)} dx$, where Γ is any path that encloses i and 0 but not $-i$.