King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Final Exam for Math 201 (071)

Date: January 22, 2008 Instructor: Dr. Muhammad Yousuf

Name:.....ID #:.....Sec # 04 06

Time Allowed: 150 Minutes

NOTE: Show complete and clear work to get full credit. Calculator is not allowed.

Q.1: Write the equation $\rho^2\left(\sin^2\phi - 3\cos^2\phi\right) = 1$ in cartesian coordinates. **(10 pts)**

Q.2: Find symmetric equation of the line of intersection of the planes $x + y - z = 2$ and $3x - 4y + 5z = 6$. Also find angle between these planes. **(5+5 pts)**

Q.3: Show that $f(x,y) = xe^{xy}$ is differentiable at the point $P(1,0)$ and find the linearization $L(x,y)$ of $f(x,y)$ at the point $P(1,0)$. Use $L(x,y)$ to approximate $f(1.1,-0.1)$. **(10 pts)**

Q.4: (a)Find the maximum rate of change of $f(x,y,z) = \tan(x+2y+3z)$ at the point $(-5,1,1)$. **(5 pts)**

(b) Write equations of the tangent plane and normal line to the surface $f(x,y,z) = 0$ at $(-5,1,1)$. **(5 pts)**

Q.5: Use Lagrange Multipliers to find the maximum and minimum values of the function $f(x, y, z) = 8x + 6y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 26$. **(10 pts)**

Q.6: Find volume of the solid enclosed by the surface $z = e^y \sin x + e^x \cos y$ and the planes $x = 0,\ x = \pi,\ y = 0,\ y = \dfrac{\pi}{2}$, and $z = 0$. **(10 pts)**

Q.7: Evaluate the integral $\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos x \sqrt{1+\cos^2 x}\, dxdy$ (Hint: Change order of integration). **(10 pts)**

Q.8: Use double integrals in polar coordinates to find the area bounded by the circles $r = \sin\theta$ and $r = \cos\theta$. **(12 pts)**

Q.9: Find, if any, the local maximum, local maximum and saddle point(s) of the function $f(x, y) = 2x^4 + 2y^4 - 8xy + 12$. **(12 pts)**

Q:10: Use chain rule to find $\dfrac{\partial z}{\partial r},\ \dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ if $z = w\sin^{-1}(uv)$, and $u = r+s,\ v = s+t,\ w = t+r$. **(12 pts)**

Q.11: Find volume of the region bounded by the surfaces $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$. **(12 pts)**

Q.12: Evaluate $\iiint_E x^2 dV$, where E is the solid region bounded by the hemispheres $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{9 - x^2 - y^2}$and the $xy - plane$. **(12 pts)**