

Math 201

Maple Handout # 10.4

Area and Arc Lengths in Polar Coordinates

In this assignment we will learn how to find area enclosed by polar curves.

Whenever you open a Maple file, press ENTER with cursor anywhere on **restart:** and on **with(plots):**

NOTE: To type click on T icon. To insert > for typing math, click on [> icon

```
> restart;
```

```
> with(plots):
```

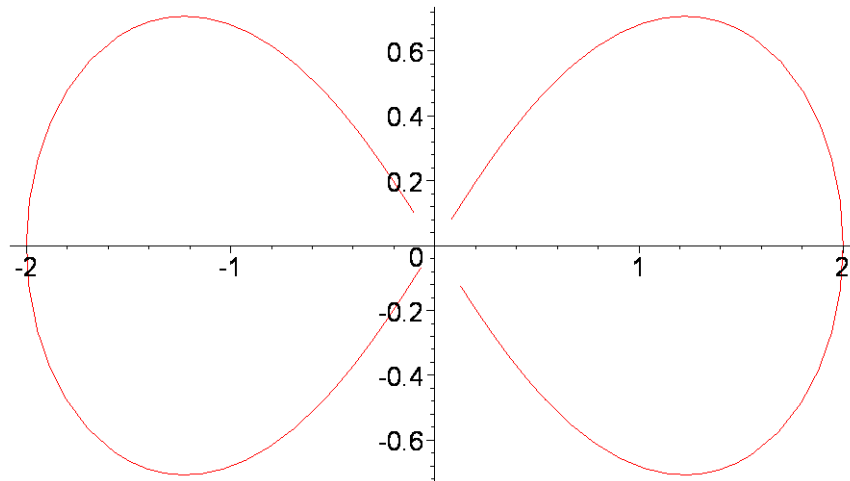
```
Warning, the name changecoords has been redefined
```

Problem # 11

```
> r(t):=sqrt(4*cos(2*t));
```

$$r(t) := 2\sqrt{\cos(2t)}$$

```
> plot([r(t),t,t=0..2*Pi],coords=polar);
```



We evaluate the integral

$$2 \int_0^{\pi/4} r(t)^2 dt$$

```
> 4*1/2*int(r(t)^2,t=0..Pi/4);
```

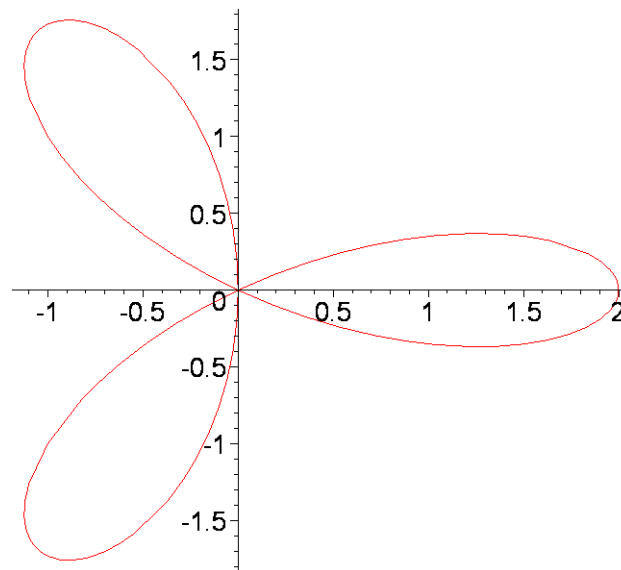
4

Problem # 13

```
> r1(t):=2*cos(3*t);
```

$$r1(t) := 2 \cos(3t)$$

```
> plot([r1(t),t,t=0..Pi],coords=polar);
```



We evaluate the integral

$$3 \int_0^{\frac{\pi}{6}} r_1(t)^2 dt$$

```
> 1/2*6*int(r1(t)^2,t=0..Pi/6);
```

π

Problem # 29

```
> r1(t):=sin(t); r2(t):=cos(t);
```

$r_1(t) := \sin(t)$

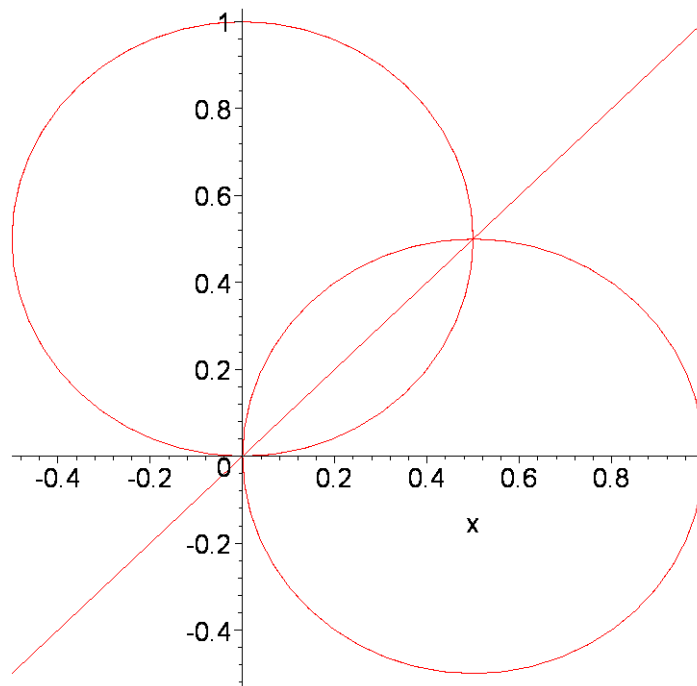
$r_2(t) := \cos(t)$

```
> P1:=plot([r1(t),t,t=0..2*Pi],coords=polar):
```

```
> P2:=plot([r2(t),t,t=0..2*Pi],coords=polar):
```

```
> P3:=plot(x,x=-.5..1):
```

```
> display(P1,P2,P3);
```



We will evaluate the integral

$$.5 \int_0^{\frac{\pi}{4}} \sin(t)^2 dt + .5 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(t)^2 dt$$

```
> 0.5*(int(sin(t)^2,t=0..Pi/4))+0.5*(int(cos(t)^2,t=Pi/4..Pi/2));
      -0.2500000000 + 0.1250000000 π
```

```
>
```

Problem # 33

```
> r1(t):=3+2*sin(t); r2(t):=2;
```

```
      r1(t) := 3 + 2 sin(t)
```

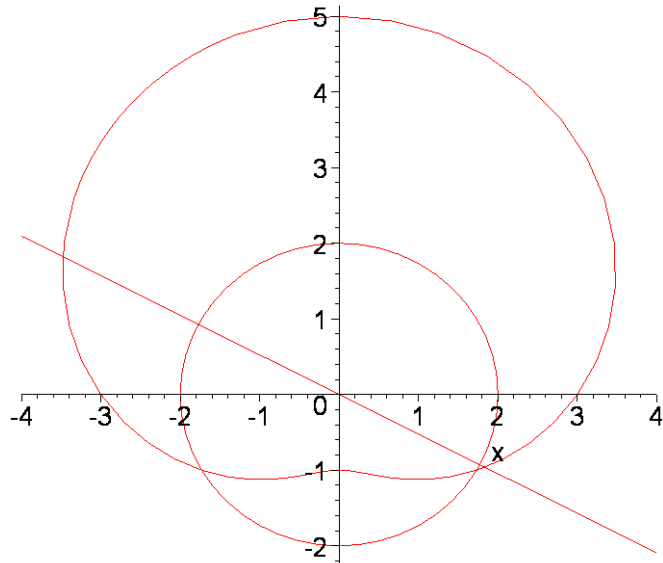
```
      r2(t) := 2
```

```
> P1:=plot([r1(t),t,t=0..2*Pi],coords=polar):
```

```
      P2:=plot([r2(t),t,t=0..2*Pi],coords=polar):
```

```
      P3:=plot(-Pi/6*x,x=-4..4):
```

```
      display(P1,P2,P3);
```



```
> solve(3+2*sin(t)=2,t);
```

$$-\frac{\pi}{6}$$

$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} r_1(t)^2 dt + \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} r_2(t)^2 dt$$

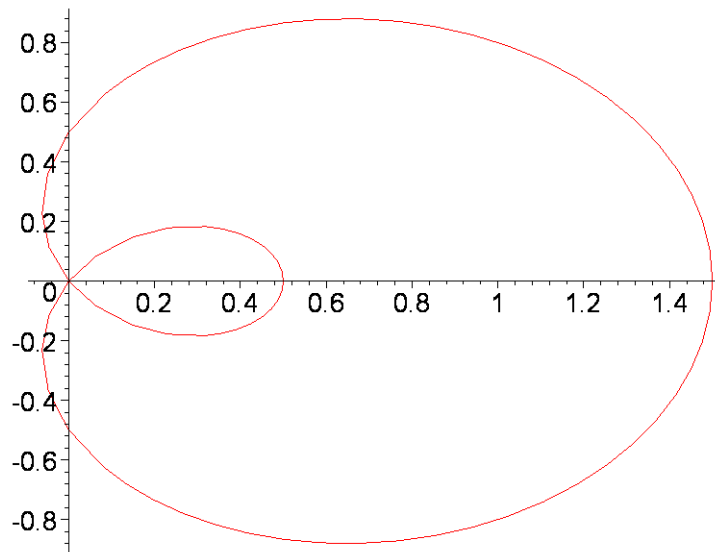
```
> 2*(1/2*int(r1(t)^2,t=-Pi/2..-Pi/6)+1/2*int(r2(t)^2,t=-Pi/6..Pi/2))
;
```

$$\frac{19\pi}{3} - \frac{11\sqrt{3}}{2}$$

```
> r(t):=1/2+cos(t);
```

$$r(t) := \frac{1}{2} + \cos(t)$$

```
> plot([r(t),t,t=0..2*Pi],coords=polar);
```



Evaluate the integral

$$\int_0^{\frac{2\pi}{3}} r(t)^2 dt - \int_{\frac{2\pi}{3}}^{\pi} r(t)^2 dt$$

```
> 2*(1/2*int(r(t)^2,t=0..2*Pi/3)-1/2*int(r(t)^2,t=2*Pi/3..Pi));
```

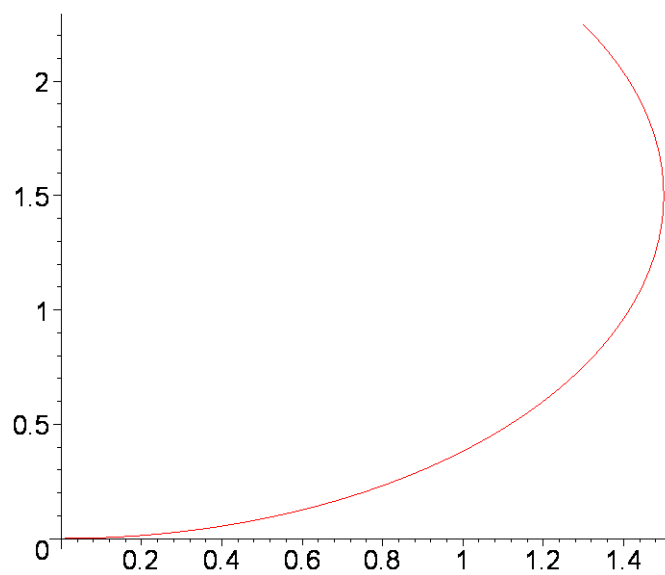
$$\frac{3\sqrt{3}}{4} + \frac{\pi}{4}$$

Problem # 45

```
> r(t):=3*sin(t);
```

$$r(t) := 3 \sin(t)$$

```
> plot([r(t),t,t=0..Pi/3], coords=polar);
```



To find the arc length, we evaluate the integral $\int_0^{\frac{\pi}{3}} \sqrt{r(t)^2 + \left(\frac{d}{dt}r(t)\right)^2} dt$

```
> int(sqrt(r(t)^2+diff(r(t),t)^2),t=0..Pi/3);
```

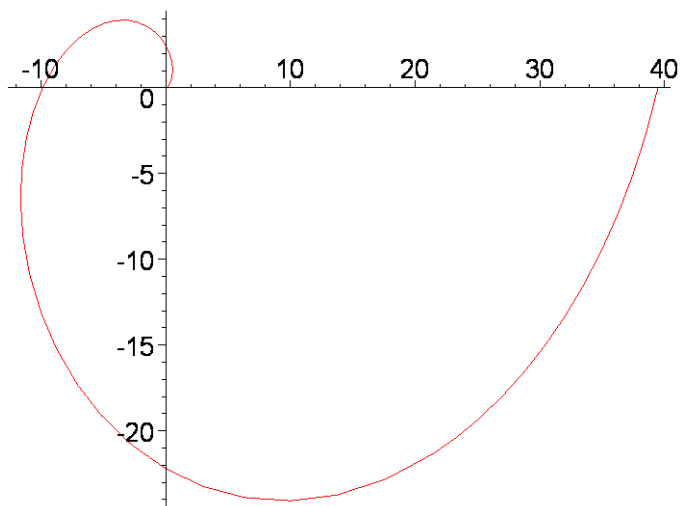
π

Problem#48

```
> r(t):=t^2;
```

$$r(t) := t^2$$

```
> plot([r(t),t,t=0..2*Pi],coords=polar);
```



We evaluate the integral $\int_0^{2\pi} \sqrt{r(t)^2 + \left(\frac{d}{dt}r(t)\right)^2} dt$

```
> int(sqrt(r(t)^2+diff(r(t),t)^2),t=0..2*Pi);
```

$$2 \left(-\frac{4\pi^{(7/2)} \left(1 + \frac{1}{\pi^2}\right)^{(3/2)}}{3} + \frac{4\sqrt{\pi}}{3} \right)$$

$-\frac{\quad}{\sqrt{\pi}}$

```
> evalf(%);
```

92.89623754

```
>
```