Math 201
Maple Handout # 10.2
Calculus with Parametric Curves

NOTE: To type click on T icon. To insert > for typing math, click on [> icon
Whenever you open a Maple file, press ENTER with cursor anywhere on **restart:** and on **with(plots):**

```
restart:
```

This command clears maple memory and assign new values to variable.

```
> restart:
> with(plots):
```

This command includes maple inbuilt package "plots"

```
Warning, the name changecoords has been redefined
```

Define a parametric curve

```
> f(t):=r*(t+sin(t)); g(t):=r*(1+cos(t));
```

$$f(t) := r\,(t + \sin(t))$$
$$g(t) := r\,(1 + \cos(t))$$

$$\frac{d}{dt} f(t)$$

```
> diff(f(t),t);
```

$$r\,(1 + \cos(t))$$

```
> diff(g(t),t);
```

$$-r\,\sin(t)$$

$$Area := \int_0^{2\pi} g(t)\left(\frac{d}{dt} f(t)\right) dt$$

Area under a Parametric Curve $\int_0^{2\pi} g(t)\left(\frac{d}{dt} f(t)\right) dt$

```
> Area:=int(g(t)*diff(f(t),t),t=0..2*Pi);
```

$$Area := 3\,r^2\,\pi$$

Arc Length $\int_0^{2\pi} \sqrt{\left(\frac{d}{dt} f(t)\right)^2 + \left(\frac{d}{dt} g(t)\right)^2}\, dt$

```
> Arc:=int(sqrt(diff(f(t),t)^2+diff(g(t),t)^2),t=0..2*Pi);
```

$$Arc := 8\, r\, \mathrm{csgn}(r)$$

Surface Area of revolution about x-axis

$$2\,\pi \int_0^{\pi} \mathrm{g}(t) \sqrt{\left(\frac{d}{dt}\mathrm{f}(t)\right)^2 + \left(\frac{d}{dt}\mathrm{g}(t)\right)^2}\, dt$$

```
> f(t):=r*cos(t); g(t):=r*sin(t);
```

$$\mathrm{f}(t) := r\cos(t)$$

$$\mathrm{g}(t) := r\sin(t)$$

```
> SA:=2*Pi*int(g(t)*sqrt(diff(f(t),t)^2+diff(g(t),t)^2),t=0..Pi);
```

$$SA := 4\,\pi\, r^2\, \mathrm{csgn}(r)$$

Exercise 10.2

Prob#6

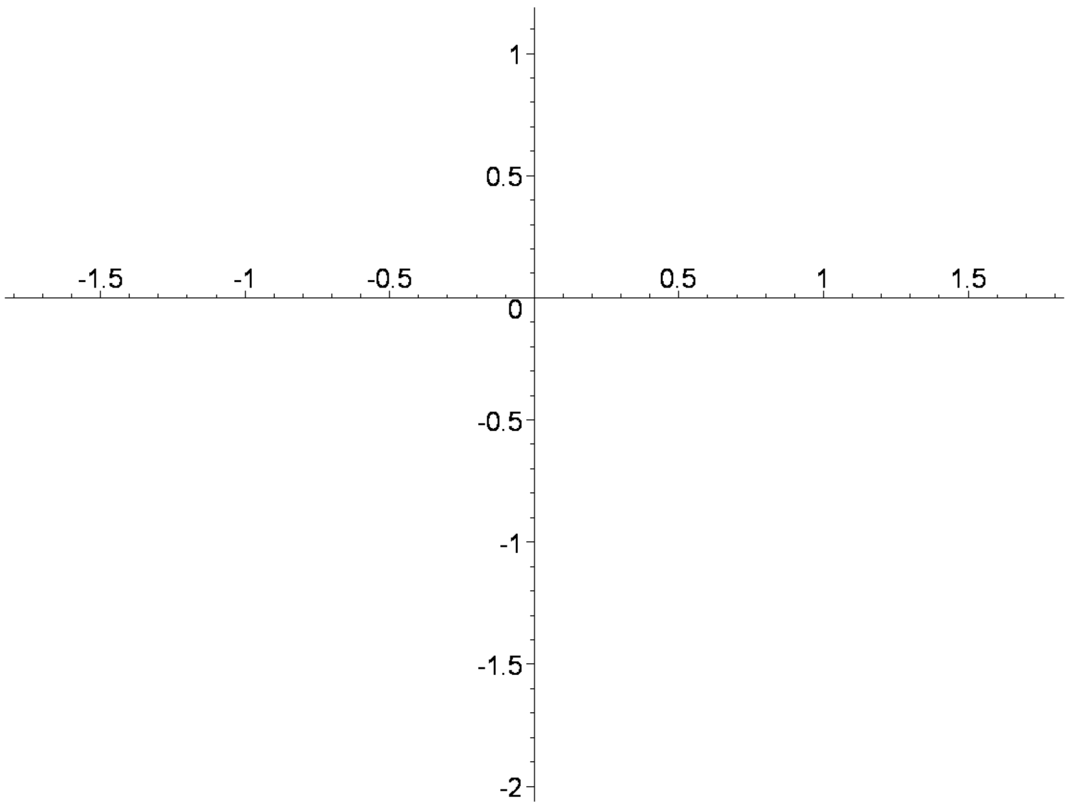
```
> f(t):=cos(t)+sin(2*t);
```

$$\mathrm{f}(t) := \cos(t) + \sin(2\,t)$$

```
> g(t):=sin(t)+cos(2*t);
```

$$\mathrm{g}(t) := \sin(t) + \cos(2\,t)$$

```
> animatecurve([f(t),g(t),t=0..2*Pi],frames=200,numpoints=100);
```



Derivative $\frac{dy}{dx} = \frac{\frac{d}{dt}\,\mathrm{g}(t)}{\frac{d}{dt}\,\mathrm{f}(t)}$

```
> d1:=diff(g(t),t)/diff(f(t),t);
```

$$d1 := \frac{\cos(t) - 2\sin(2\,t)}{-\sin(t) + 2\cos(2\,t)}$$

Substitute t = 0 in the derivative

```
> m:=evalf(subs(t=0,d1));
```

$$m := 0.5000000000$$

Substitute t = 0 in *f(t)* and *g(t)*

```
> x1:=evalf(subs(t=0,f(t)));
```

$$x1 := 1.$$

```
> y1:=evalf(subs(t=0,g(t)));
```

$$y1 := 1.$$

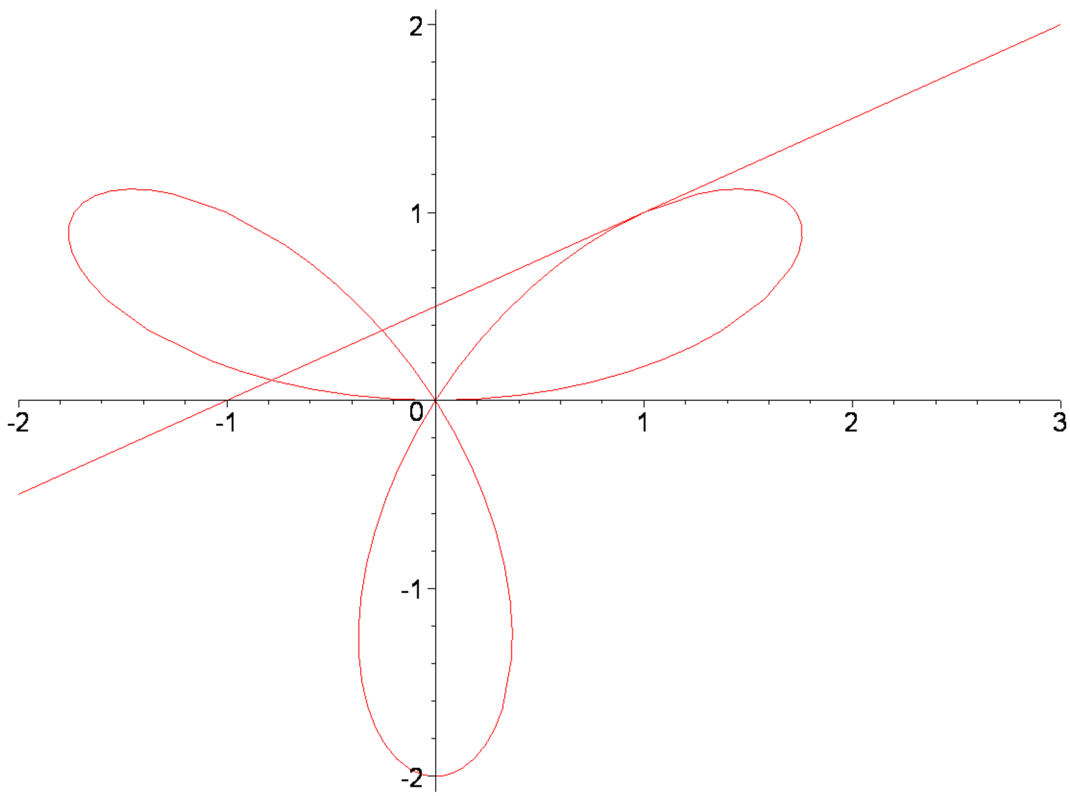
Equation of the tangent line: $(y - y1) = m\,(x - x1)$

```
> y:=x->m*(x-x1)+y1;
>
```

$$y := x \to m\,(x - x1) + y1$$

Compute two plots and display in the same window

```
> p1:=plot([f(t),g(t),t=0..2*Pi]):
> p2:=plot(y(x),x=-2..3):
> display(p1,p2);
```



>

Prob#9

```
> x1:=t->2*sin(2*t); y1:=t->2*sin(t);
```

$$x1 := t \rightarrow 2\sin(2\,t)$$

$$y1 := t \rightarrow 2\sin(t)$$

```
> diff(x1(t),t);
```

$$4\cos(2\,t)$$

```
> diff(y1(t),t);
>
```

$$2\cos(t)$$

```
> solve(x1(t)=sqrt(3),t);
```

$$\frac{\pi}{6}$$

```
> solve(y1(t)=1,t);
```

$$\frac{\pi}{6}$$

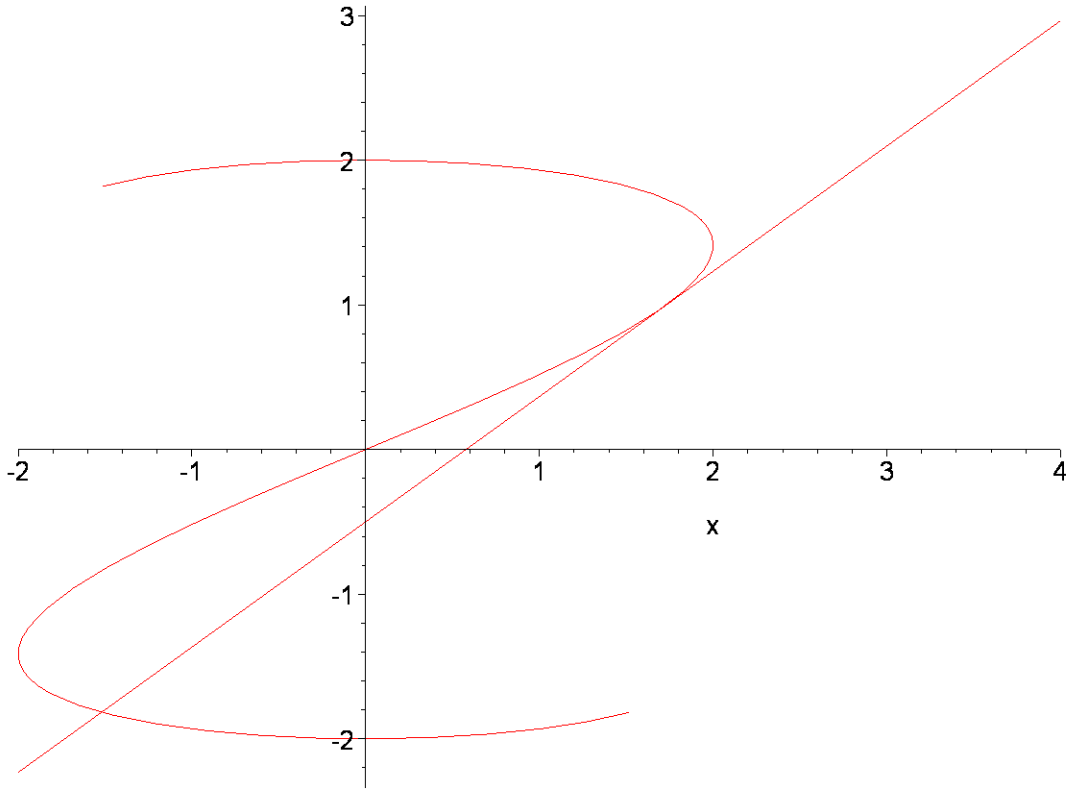
```
> m:=subs(t=Pi/6,(diff(y1(t),t)/diff(x1(t),t)));
```

$$m := \frac{1}{2}\,\frac{\cos\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{3}\right)}$$

```
> y:=x->m*(x-sqrt(3))+1;
```

$$y := x \to m\,(x - \sqrt{3}) + 1$$

```
> Plt1:=plot(y(x),x=-2..4):
> Plt2:=plot([x1(t),y1(t),t=-2..2]):
  display(Plt1,Plt2);
```




```
>
>
```