King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Exam # 2, Math 302 – 03

Name:... .....Serial #:..........................

Note: Show all your work to get full credit.

Q.1: Let $x(t) = \sin(t),\ y(t) = \cos(t),\ z(t) = t,\ 0 \le t \le 2\pi$. Write the position vector and the tangent vector for the curve. Also find the length function.

Q.2: Find the directional derivative of the function $\varphi(x, y, z) = 1 - x^2 - y^2 - xyz$ in the direction of the vector $\mathbf{u} = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Also find equation of the tangent plane to the surface $\varphi(x, y, z) = 0$, at the point $(1, 1, -1)$.

Q.3: Let $\mathbf{F} = \sinh(x^2 - z)\mathbf{i} + 2xy\mathbf{j} + (z^2 - y^2)\mathbf{k}$. Find Divergence of $\mathbf{F}$ and Curl of $\mathbf{F}$.

Q.4: Sow that $\nabla \times (\nabla\varphi) = \mathbf{0}$, and $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

Q.5: Evaluate the integral $\int\limits_C \mathbf{F} \cdot d\mathbf{R}$, where $\mathbf{F} = \sin(x)\mathbf{i} + 2z\mathbf{j} - \mathbf{k}$ and C is given by $x = t,\ y = t^2,\ z = t^3$, for $0 \leq t \leq 3$.

Q.6: Use Green's Theorem to evaluate the integral $\oint\limits_C \mathbf{F} \cdot d\mathbf{R}$, where $\mathbf{F} = x^2y\mathbf{i} - xy^2\mathbf{j}$ and C is the boundary of the region $x^2 + y^2 \leq 4,\ y \geq 0$.

Q.7: Evaluate the surface integral $\iint\limits_{\Sigma} f(x,y,z)\,d\sigma$, where $f(x,y,z) = z$ and Σ is the cone $x = \sqrt{y^2+z^2}$, for $y^2+z^2 \le 9$.

Q.8: Evaluate the surface integral $\iint\limits_{\Sigma} f(x,y,z)\,d\sigma$, where $f(x,y,z) = x$ and Σ is part of the plane $5x+2y+z=10$ in the first octant.

Q.9: Evaluate $\oint\limits_C \mathbf{F} \cdot d\mathbf{R}$, where $\mathbf{F} = (-4\cosh(xy) - 4xy\sinh(xy))\,\mathbf{i} - 4x^2\sinh(xy)\mathbf{j}$, and C is any path from $(1,1)$ to $(2,2)$. (Hint: find potential function φ for $\mathbf{F}$ if it is conservative)

Q.10: Calculate one side of the Gauss's Divergence Theorm for the vector field $\mathbf{F} = (x-y)\,\mathbf{i} + (y-4xz)\,\mathbf{j} + xz\,\mathbf{k}$, Σ is the rectangular box bounded by the coordinate planes $x=0,\ y=0,\ z=0$, and by the planes $x=3,\ y=4,\ z=5$.