King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Midterm Exam 2 for Math 201 (071)

Date: December 02, 2007 Instructor: Dr. Muhammad Yousuf

Name:...ID #:................................Sec # 04 06

Time Allowed: 100 Minutes

NOTE: Show complete and clear work to get full credit. Calculator is not allowed.

Q.1: Identify the quadratic surface $4x^2 + 6y^2 - 9z^2 - 8x + 24y + 18z = 17$. Write what are (names only) its horizontal and vertical traces. **(10 pts)**

Q.2: Two surfaces are tangent to each other at a point if they have the same tangent plane at that point. Show that the two surfaces $3x^2 + 2y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$ are tangent to each other at $(1, 1, 2)$. **(10 pts)**

Q.3: Let $f(x, y) = \log_{10}(x + 2y - 1)$

(a) Write and sketch the domain of f. **(4 pts)**

(b) Evaluate $f(1, 50)$ and $f\left(\sqrt{10}, \frac{1}{2}\right)$. **(4 pts)**

(c) Write the range of f. **(2 pts)**

Q.4: (a) Write the cartesian equation for the surface whose spherical equation is $\rho = \sin\theta\cos\phi$. **(4 pts)**

(b) Write the spherical equation for the surface whose cartesian equation is $x^2 - y^2 - 2z^2 = 4$. **(3 pts)**

(c) Find the limit using polar coordinates $\lim_{(x,y)\to(0,0)} \cos^{-1}\left(\frac{x^3 - xy^2}{x^2 + y^2}\right)$. **(3 pts)**

Q.5: Explain why $f(x,y) = 3\sin x + 4\cos y$ is differentiable at the point $P\left(0, \frac{\pi}{2}\right)$. Also find the linearization $L(x,y)$ of $f(x,y)$ at the point P. **(10 pts)**

Q.6: Find the partial derivatives $f_{xx},\ f_{yy},\ f_{xy}$ for the function $f(x,y) = \tan(3x+2y)$. **(10 pts)**

Q.7: Use chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = \tan^{-1}\left(x^2y^3\right), \ x = \ln\left(s^3t\right), \ y = \sec\left(s+t\right).$ **(10 pts)**

Q.8: The length l, width w and height h of a box are changing with time. If l is increasing at a rate of 3 m/s, w is decreasing at a rate of 2 m/s, and h is increasing at a rate of 1 m/s. Find the rate of change of the volume of the box when $l = 10$, $w = 8$, and $h = 5$. **(10 pts)**

Q.9: Find the directional derivative of the function $f(x, y, z) = \dfrac{1}{2x + 3y - z}$ at the point $(1, 2, 3)$ in the direction $v = \langle 1, 2, 3 \rangle$. **(10 pts)**

Q.10: If $u = f(x, y)$, where $x = e^s \cos t$ and $y = e^s \sin t$. Show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = e^{-2s} \left[\dfrac{\partial^2 u}{\partial s^2} + \dfrac{\partial^2 u}{\partial t^2} \right]$. **(10 pts)**