

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Midterm Exam 1 for Math 201 (071)

Date: October 28, 2007

Instructor: Dr. Muhammad Yousuf

Name:.....ID #:.....Sec # 04 06

Time Allowed: 90 Minutes

NOTE: Show complete and clear work to get full credit. Calculator is not allowed.

Q.1: Let parametric equations of a curve are: $x(t) = 2 + 3t^2$ and $y(t) = 3 + 2t^3$.

(a) Find $\frac{dx}{dt}$ (1 pts)

(b) Find $\frac{dy}{dt}$ (1 pts)

(c) Find $\frac{dy}{dx}$ (2 pts)

(d) Find $\frac{d^2y}{dx^2}$ (3 pts)

(e) Find length of the curve for $0 \leq t \leq 1$. (8 pts)

Q.2: Sketch graph of the polar curve $r = 2 + 2 \sin(\theta)$. Show the points on the curve for $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$. (4 pts)

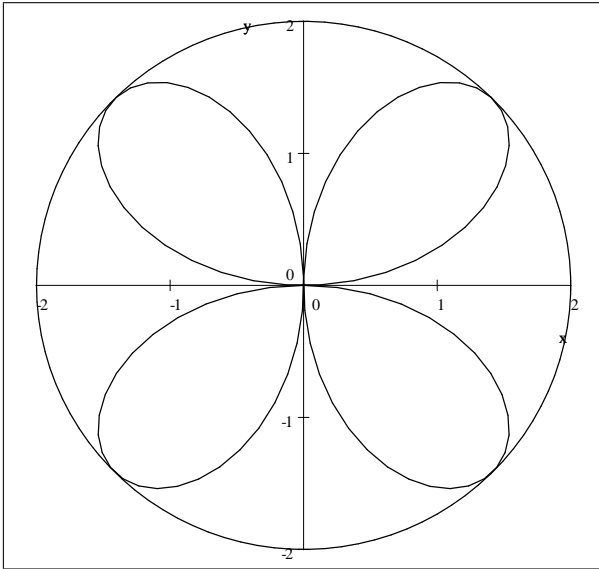
(a) Find $\frac{dx}{d\theta}$ (2 pts)

(b) Find $\frac{dy}{d\theta}$ (2 pts)

(c) Find $\frac{dy}{dx}$ (2 pts)

(d) Find equation of tangent line at $\theta = \frac{\pi}{4}$ (5 pts)

Q.3: Find area of the region that lies inside $r = 2$ and outside $r = 2 \sin(2\theta)$. (10 pts)



Q.4: If $r = \langle x, y, z \rangle$, $a = \langle 2, 3, 1 \rangle$, and $b = \langle -2, 1, 5 \rangle$. Show that the vector equation $(r - a) \cdot (r - b)$ represents a sphere. Find center and radius of the sphere (10 pts)

Q.5: Find scalar and vector projection of the vector $\mathbf{u} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ in the direction of $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. (5+5 pts)

Q.6: Determine whether the three vectors $\mathbf{u} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, and $\mathbf{w} = 0\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}$ are coplanar or not. Show all your work. (10 pts)

Q.7: Find equation of a line containing the point $(1, 2, -3)$ and is perpendicular to the plane $3x - 2y + z = 5$. Write your answer in symmetric and parametric form. (10 pts)

Q.8: Find equation of a plane passing through the points $(1, 2, 3)$, $(-3, 4, 2)$, and $(2, 4, 5)$. (10 pts)

Q.9: Find angle between the planes $3x - 2y + z = 5$ and $2x + y - 4z = 5$. (5 pts)

Q.10: Prove that $(\mathbf{u} - \mathbf{v}) \times (\mathbf{u} + \mathbf{v}) = 2(\mathbf{u} \times \mathbf{v})$ (5 pts)