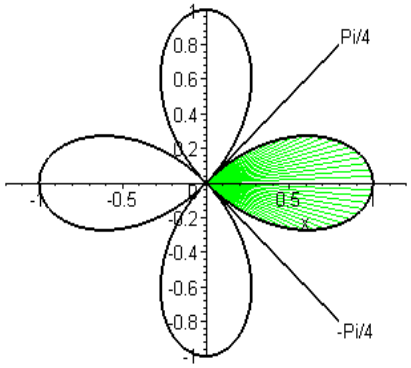
Area Bounded by Polar Curves

P.1: Area enclosed by one loop of $r = \cos(2\theta)$, for $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$.

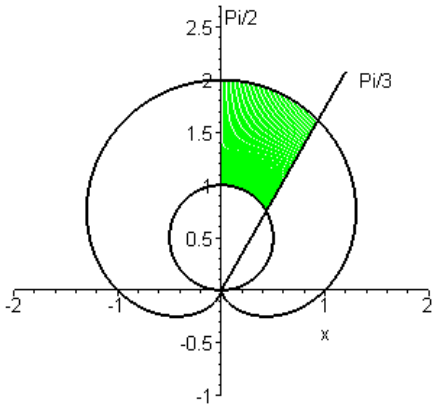
Sol:



$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}\cos^2(2\theta)\, d\theta = \frac{1}{8}\pi$$

P.2: Area inside $r = 1 + \sin(\theta)$ and outside $r = \sin(\theta)$ for $\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}$.

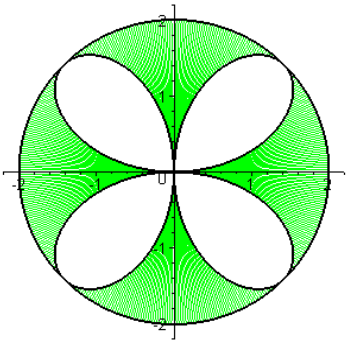
Sol:



$$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}\left((1+\sin(\theta))^2 - \sin^2(\theta)\right) d\theta = \frac{1}{12}\pi + \frac{1}{2}$$

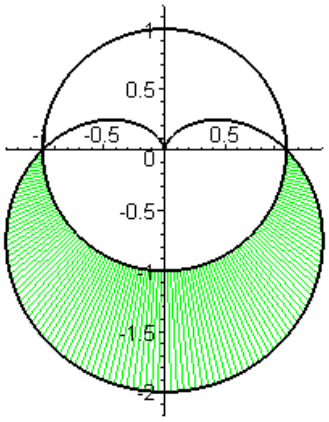
P.3: Area inside $r = 2$ and outside $r = 2\sin(2\theta)$.

Sol:



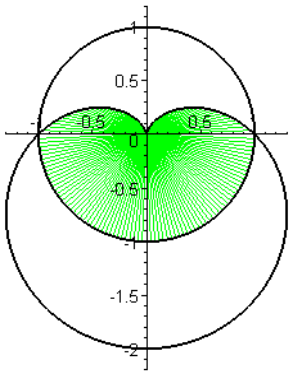
$$A = 4\pi - 4\int_{0}^{\frac{\pi}{2}} \frac{1}{2}\left(4\sin^2(2\theta)\right) d\theta = 2\pi.$$

P.4: Area inside $r = 1 - \sin(\theta)$ and outside $r = 1$.
Sol:



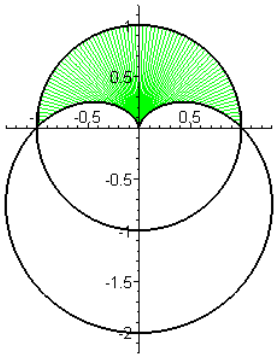
$$A = \int_{\pi}^{2\pi} \frac{1}{2}\left((1-\sin(\theta))^2 - 1\right) d\theta = \frac{1}{4}\pi + 2.$$

P.5: Area inside both $r = 1 - \sin(\theta)$ and $r = 1$.
Sol:



$$A = \int_{0}^{\pi} \frac{1}{2}(1-\sin(\theta))^2 \, d\theta + \int_{\pi}^{2\pi} \frac{1}{2}(1) \, d\theta = \frac{5}{4}\pi - 2.$$

P.6: Area inside $r = 1$ and outside $r = 1 - \sin(\theta)$.
Sol:



$$A = \int_{0}^{\pi} \frac{1}{2}\left(1 - (1-\sin(\theta))^2\right) d\theta = 2 - \frac{1}{4}\pi.$$