Math 202 Quiz # 6

Name: Solution **Sr. #** ________

1. Find a linear differential operator that annihilates each one of the following functions:

a) $f(x)=(5-e^x)^2$

$f(x)=25-10e^x+e^{2x}$

$D \quad D-1 \quad D-2$

$\therefore \text{Ann}[f(x)]=D(D-1)(D-2)$

b) $g(x)=x^2\cos 3x+e^{2x}\sin 3x$

$(D^2+9)^3 \qquad D^2-4D+13$

$\text{Ann}=\left[D^2-2\alpha D+\alpha^2+\beta^2\right]^n$

$\therefore \text{Ann}[g(x)]=(D^2+9)^3(D^2-4D+13)$

2. Solve the following DE by undetermined coefficients: $y''+y=8\cos 2x-4\sin x$ ---- (1)

First we solve $y''+y=0$

$\Rightarrow \lambda^2+1=0 \Rightarrow \lambda=\pm i \Rightarrow y_h=c_1\cos x+c_2\sin x$

Write DE(1) as: $(D^2+1)y=8\cos 2x-4\sin x$ ------- (2)

$\text{Ann}[8\cos 2x-4\sin x]=(D^2+4)(D^2+1)$

Apply $(D^2+4)(D^2+1)$ on both sides of (2):

$(D^2+4)(D^2+1)(D^2+1)y=0$

$(D^2+4)(D^2+1)^2y=0$ ---------- (3)

Solving (3): $(\lambda^2+4)(\lambda^2+1)^2=0 \Rightarrow \lambda=\pm i, \pm i, \pm 2i$

$$y=\underbrace{c_1\cos x+c_2\sin x}_{y_h}+\underbrace{c_3x\cos x+c_4x\sin x+c_5\cos 2x+c_6\sin 2x}_{y_p}$$

$\therefore y_p=Ax\cos x+Bx\sin x+C\cos 2x+D\sin 2x$

$y_p'=-Ax\sin x+A\cos x+Bx\cos x+B\sin x-2C\sin 2x+2D\cos 2x$

$y_p''=-Ax\cos x-A\sin x-A\sin x-Bx\sin x+B\cos x+B\cos x-4C\cos 2x-4D\sin 2x$

Substitute in (1): $\cancel{-Ax\cos x}-A\sin x-A\sin x\cancel{-Bx\sin x}+B\cos x+B\cos x-4C\cos 2x-4D\sin 2x$

$+\cancel{Ax\cos x}+\cancel{Bx\sin x}+C\cos 2x+D\sin 2x=8\cos 2x-4\sin x$

$\Rightarrow \quad -3C\cos 2x-2A\sin x+2B\cos x-3D\sin 2x=8\cos 2x-4\sin x$

$\Rightarrow \quad -3C=8 \Rightarrow \boxed{C=-\frac{8}{3}},\ -2A=4\Rightarrow \boxed{A=2},\ \boxed{B=0},\ \boxed{D=0}$

$\therefore \quad y_p=2x\cos x-\frac{8}{3}\cos 2x$

$y=y_h+y_p=c_1\cos x+c_2\sin x+2x\cos x-\frac{8}{3}\cos 2x$