

Math 202 – Quiz # 2c

Name: _____

Ser. # _____

Solve the following DE by using two different methods: $(x^2+4)y' + 3xy = x$

Method 1: (Linear)

$$(x^2+4)\frac{dy}{dx} + 3xy = x$$

$$\frac{dy}{dx} + \frac{3x}{x^2+4}y = \frac{x}{x^2+4} \quad (\text{Linear DE})$$

$$P(x) = e^{\int \frac{3x}{x^2+4} dx} = e^{\frac{3}{2} \int \frac{2x}{x^2+4} dx} = e^{\frac{3}{2} \ln(x^2+4)} = (x^2+4)^{\frac{3}{2}}$$

$$\frac{d}{dx} \left[y (x^2+4)^{\frac{3}{2}} \right] = \frac{x}{x^2+4} (x^2+4)^{\frac{3}{2}} = x (x^2+4)^{\frac{1}{2}}$$

$$\Rightarrow y (x^2+4)^{\frac{3}{2}} = \int x (x^2+4)^{\frac{1}{2}} dx = \frac{1}{3} (x^2+4)^{\frac{3}{2}} + C$$

$$y = \frac{1}{3} + C (x^2+4)^{-\frac{3}{2}}$$

Method 2: (Separating the variables)

$$(x^2+4)\frac{dy}{dx} + 3xy = x$$

$$\frac{dy}{dx} = \frac{x}{x^2+4} - \frac{3xy}{x^2+4} = \frac{x}{x^2+4} [1-3y] \quad (\text{separable})$$

$$\frac{dy}{1-3y} = \frac{x}{x^2+4} dx$$

$$-\frac{1}{3} \ln|1-3y| = \frac{1}{2} \ln(x^2+4) + \ln|K|$$

$$\frac{1}{2} \ln(x^2+4) + \frac{1}{3} \ln|1-3y| + \ln|K| = 0$$

$$\ln \left| (x^2+4)^{\frac{1}{2}} (1-3y)^{\frac{1}{3}} K \right| = 0$$

$$K \sqrt{x^2+4} \sqrt[3]{1-3y} = 1 \Rightarrow \sqrt[3]{1-3y} = \frac{1}{K(x^2+4)^{\frac{3}{2}}}$$

$$1-3y = \frac{1}{K^3 (x^2+4)^{\frac{3}{2}}}$$

$$\therefore y = \frac{1}{3} + C (x^2+4)^{-\frac{3}{2}}$$