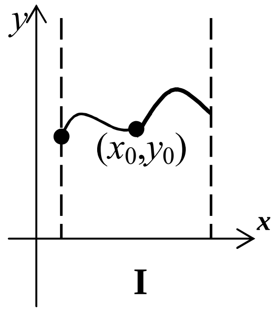
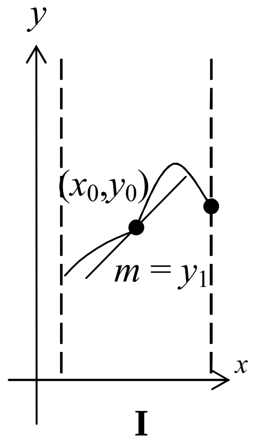
1.2 Initial Value Problem (IVP)

[An ODE with Given Relation(s) between value of x and value of y (and its Derivatives)]

Type	Problem	Explanation	Geometric Meaning	Graph
1^{st} Order I V P	$y' = f(x, y)$ …(I) Subject to $y(x_0) = y_0$ …(II)	i. **(I)** gives a One-parameter family of solutions $g(x, y, c) = 0$ defined on an Interval **I**. ii. Condition **(II)** when used in the Solution of **(I)** gives the value of constant c.	The solution curve of the ODE Passes through the Point (x_0, y_0).	
2^{nd} Order I V P	$y'' = f(x, y)$ …(I) Subject to $y(x_0) = y_0$ $y'(x_0) = y_1$ …(II)	i. **(I)** gives a Two-parameter family of solutions $g(x, y, c_1, c_2) = 0$ ii. Two Initial Conditions given by **(II)** when used in the Solution of **(I)** gives the value of constants c_1 and c_2. Slope m of Tangent Line to the solution curve at (x_0, y_0) is y_1.	i. The solution curve of the ODE Passes through the Point (x_0, y_0). ii. Slope of Tangent Line to the solution curve at (x_0, y_0) is y_1	

Example 1.

Solve $y' = 2x$ subject to $y(1) = -3$

Solution: $y = x^2 + c$. **I C** $\Rightarrow -3 = 1 + c$

Ans: $y = x^2 - 4$

Theorem: Sufficient Condition for Existence & Uniqueness of Solution for IVP

Given Condition 'Sufficient'	1. Region $R = \{(x,y): a \le x \le b;\ c \le x \le d\}$. 2. (x_0,y_0) is point contained inside R 3. $f(x, y)$ and $f_y(x, y)$ are continuous on R.

<u>**Conclusion**</u>: The IVP $y' = f(x, y)$ Subject to $y(x_0) = y_0$ has a Unique Solution in an interval **I** containing x_0.

Example: i. Both functions are the solutions of the IVP: $y' = y\sqrt{x}$ Subject to $y(0) = 0$ (Check!)

ii. $f(x, y)$ and $f_y(x, y)$ are continuous on

$R = \{(x,y):\ -\infty < x < \infty;\ 0 < x < \infty\}$. (Check!)

[Note: $(0, 0) \notin R$]

iii. Explain why the IVP: $y' = y\sqrt{x}$ Subject to $y(0) = 2$ has a unique solution on an interval centered at 0.

Exercise: Determine a Region R in xy-plane in which the ODE $(y^2 - x^2) - y' = y^2$ has a unique solution through a point $(x_0, y_0) \in R$.

Example 3. Show that

$y = c_1\cos 4x + c_2\sin 4x$

is a two-parameter family of solutions of

$y'' + 16y = 0$

Example 4. Find the Solution of the IVP

$y'' + 16y = 0$

subject to $y(\pi/2) = -2;\ y'(\pi/2) = 1$.

Solution: Consider the solution

$y = c_1\cos 4x + c_2\sin 4x$ (**Example 3**)

Use the given Initial Conditions to find c_1 and c_2. [Ans: $c_1 = -2$; $c_2 = ¼$]

Difference between Necessary and Sufficient Conditions

i. Given condition is Necessary & it is not Satisfied: the Conclusion will not hold.

ii. Given condition is Necessary & it is Satisfied: the Conclusion may or may not hold.

iii. Given condition is Sufficient & it is not Satisfied: the Conclusion may or may not hold.

iv. Given condition is Sufficient & it is Satisfied: the Conclusion will definitely hold hold