Questions for review on Math 260

Introduction to Differential Equations and Linear Algebra

Prepared by

Mohammad Samman

Department of Mathematics and Statistics
KFUPM

December 2007

1. State what is meant by a vector space; give an example.

2. Define each of the following:
   a) subspace
   b) basis for a vector space
   c) dimension of a vector space.

3. Show that the set *P* of all polynomials form a vector space.

4. Let $W = \{(x, y, z) \in R^3 : z = x + y\}$. Show that *W* is a subspace of R^3.

5. Let *W* be the set of all vectors (x_1, x_2, x_3, x_4) in R^4 such that $x_1 x_4 = 0$. Prove or disprove that W is a subspace of R^4.

6. Express the vector w = (2, -7, 9) as a linear combination of the vectors $v_1 = (1, -2, 2), v_2 = (3, 0, 1), v_3 = (1, -1, 2)$.

7. Determine whether the following vectors are linearly independent or not:
   (0, 0, 1,1), (-1, 1, 1,2), (1, 1, 0, 0), (2, 1, 2, 1).

8. Define each of the following:
   a) similar matrices
   b) diagonal matrix
   c) diagonalizable matrix.

9. Show that if *A* and *B* are similar matrices and *B* and C are similar matrices then *A* and *C* are similar.

10. Determine whether the matrix $B = \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is **diagonalizable** or not**.**

11. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$**.** Find a diagonalizing matrix *P* and a diagonal matrix *D* such that $P^{-1}AP = D$.

12. Find A^{10} in the above question.

13. Find B^{-1} if $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$**.**

14. Use **Gauss-Jordan Elimination Method**, to solve the system

$$\begin{aligned} s - t + u + v &= 0 \\ 2s + 2u &= 0 \\ s + t + u - v &= 0 \\ -s - 3t - u + 3v &= 0 \end{aligned}$$

15. The Augmented matrix of a non-homogeneous system $AX = B$ of linear equations has the following Reduced Form:

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 & : & 1 \\ 0 & 0 & 0 & 1 & 1 & : & 3 \\ 0 & 0 & 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

. Find **a basis** for the solution space of $AX = 0$.

16. State what is meant by Differential Equations.

17. Do you know any application for Differential Equations; give some examples.

18. If we know a solution for a given DE, is it necessarily to be unique?

19. Complete the following table

Equation	**Order**	**Linear / Nonlinear**
$y' = 10 + y^2$		
$x^2\, dy + 5xy\, dx = 0$		
$y = 2xy' + y\,(y')^2$		
$y'' + y = \tan x$		
$y'' - 5y' + 6y = 0$		
$y' + 3x(y'')^3 = \sin x$		
$y' + 3\sin x\, y'' = \cos x$		

20. Solve $x^2 \dfrac{dy}{dx} = y - xy$.

21. Solve $x\dfrac{dy}{dx} - y = x^2 \sin x$.

22. Only find the "Integrating Factor" for the linear D.E. $(\csc x)y' - xy = \tan^2 x$.

23. Solve the initial value problem $(e^x + y)dx + (2 + x + ye^y)dy = 0,\ \ y(0) = 1$.

24. Solve the initial value problem $\frac{dy}{dx} = \cos(x+y),\ y(0) = \pi/4$.

25. If $2y^2 - \sin x = c$ is a solution of an IVP $y' = f(x,y), y\left(\frac{\pi}{6}\right) = 2$ then find c.

26. Solve $x\frac{dy}{dx} - (1+x)y = xy^2$.

27. Solve $(y^2 - xy)dx + x^2 dy = 0$.

28. The Population of a Community is known to increase at a rate Proportional to the number of People present at any time. The Population of the community is doubled after 5 years and it is10,000 after 3 years. What was the initial population? What will be the Population after 10 years?

29. Find the general solution of $y'' - 4y = 2e^{2x}$.

30. Solve the boundary value problem: $y^{//} - 10y^{/} + 25y = 0,$, $y(0) = 1,\ \ y(1) = 0$.

31. Using Wronskian show that the functions 1, $1/x$ and $\log x$ are linearly independent on the interval $(0, \infty)$.

32. Show that 1, x, $\sin x$, $\cos x$ are four linearly independent solutions of the Differential Equation $y^{(4)} + y'' = 0$ on $(-\infty, \infty)$. Write the general solution for it.

33. Use the method of **Variation of Parameters** to find the general solution of the differential equation $\frac{d^2y}{dx^2} + y = \sin x$.

34. Solve the above question using the method of **Undetermined Coefficients**.

35. Consider the DE $y'' - 8y' + 15y = \sin x + xe^x + 1 - 2x^2$.

i) Find the solution of the associated homogeneous DE.
ii) How many different types of "Particular Solutions" does this DE have?
iii) Write the general form of each particular solution of the above equation.

36. It is known that $y_1(x) = x^2$ and $y_2(x) = x - 1$ are solutions of the DE:

$(x^2 - 2x)y'' + 2(1-x)y' + 2y = 0$**.** Find the general solution of

$(x^2 - 2x)y'' + 2(1-x)y' + 2y = 6(x^2 - 2x)^2$**.**

37. Find the **eigen values** of $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, and find the corresponding **eigen vectors**.

38. Solve the system

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = -4x + 3y$$

39. Solve the system $X' = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 0 \\ 6 & 3 & -8 \end{bmatrix} X$.

40. Solve the system $X' = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{bmatrix} X$.