

### Math 260 – Quiz # 5a

Name: \_\_\_\_\_ Sec. #: \_\_\_\_\_ Sr #: \_\_\_\_\_

Find a basis and the dimension of the solution space of the system:

$$x_1 + 2x_2 + x_3 - 3x_4 = 0$$

$$2x_1 + 4x_2 + 4x_3 - x_4 = 0$$

$$3x_1 + 6x_2 + 7x_3 + x_4 = 0$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & 1 & -3 & 0 \\ 2 & 4 & 4 & -1 & 0 \\ 3 & 6 & 7 & 1 & 0 \end{array} \right] \xrightarrow{\substack{-2R_1+R_2 \\ -3R_1+R_3}} \left[ \begin{array}{cccc|c} 1 & 2 & 1 & -3 & 0 \\ 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & 4 & 10 & 0 \end{array} \right]$$

$$\begin{array}{l} \frac{1}{2}R_2 \\ -2R_2+R_3 \end{array} \rightarrow \left[ \begin{array}{cccc|c} 1 & 2 & 1 & -3 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{-R_2+R_1} \left[ \begin{array}{cccc|c} 1 & 2 & 0 & -\frac{11}{2} & 0 \\ 0 & 0 & 1 & \frac{5}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad \left. \begin{array}{l} x_1, x_3 \text{ leading} \\ x_2, x_4 \text{ free} \end{array} \right\}$$

$$\Rightarrow \begin{cases} x_4 = t \\ x_2 = s \\ x_3 = -\frac{5}{2}t \\ x_1 = -2s + \frac{11}{2}t \end{cases}$$

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2s + \frac{11}{2}t \\ s \\ -\frac{5}{2}t \\ t \end{bmatrix} = \begin{bmatrix} -2s \\ s \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{11}{2}t \\ 0 \\ -\frac{5}{2}t \\ t \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} \frac{11}{2} \\ 0 \\ -\frac{5}{2} \\ 1 \end{bmatrix}$$

∴ A basis for the solution space is  $\left\{ (-2, 1, 0, 0), \left(\frac{11}{2}, 0, -\frac{5}{2}, 1\right) \right\}$

The dimension is 2.