Math 260 Quiz # 4b

Name: Solution Serial # ______

1. Write the vector $h = (2, -7, 9)$ as a linear combination of the vectors u, v, and w where $u = (1, -1, 2)$, $v = (3, 0, 1)$, $w = (1, -2, 2)$.

$$h = c_1 u + c_2 v + c_3 w$$

$$(2, -7, 9) = c_1 (1, -1, 2) + c_2 (3, 0, 1) + c_3 (1, -2, 2)$$

$$\Rightarrow \left.\begin{array}{rcrcrcr} c_1 & + & 3c_2 & + & c_3 & = & 2 \\ -c_1 & & & - & 2c_3 & = & -7 \\ 2c_1 & + & c_2 & + & 2c_3 & = & 9 \end{array}\right\}$$

Solving this system for c_1, c_2, c_3:

$$\Rightarrow c_1 = 3, \quad c_2 = -1, \quad c_3 = 2$$

$$\therefore \quad h = 3u - v + 2w$$

2. Show that the set of all (x,y,z) such that $x = y + z$ is a subspace of R^3.

Let $W = \{(x,y,z) : x = y + z\}$. We show that W is a subspace.

Let $u, v \in W$. So, $u = (y_1 + z_1, y_1, z_1)$, $v = (y_2 + z_2, y_2, z_2)$.

Then
$$\begin{aligned} u + v &= (y_1 + z_1, y_1, z_1) + (y_2 + z_2, y_2, z_2) \\ &= (y_1 + z_1 + y_2 + z_2, y_1 + y_2, z_1 + z_2) \\ &= ((y_1 + y_2) + (z_1 + z_2), y_1 + y_2, z_1 + z_2) \in W \end{aligned}$$

$$\Rightarrow u + v \in W.$$

Also, for any $c \in \mathbb{R}$,
$$\begin{aligned} cu = c(y_1 + z_1, y_1, z_1) &= (c(y_1 + z_1), cy_1, cz_1) \\ &= (cy_1 + cz_1, cy_1, cz_1) \in W \end{aligned}$$

$$\Rightarrow cu \in W.$$

Hence W is a subspace of $\mathbb{R}^3$.