Math 260 Quiz # 3b

Name: Solution I.D. # ____________ Section # ____

1. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{bmatrix}$, find $(A^T)^{-1}$, if it exists

$$A^T = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 2 & 4 & -2 \end{bmatrix}$$

$$[A^T | I] = \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 4 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3]{R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 & 1 & 0 \\ 0 & -2 & -2 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & -2 & -2 & -2 & 0 & 1 \\ 0 & 5 & 1 & 1 & 1 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 5 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & -4 & -4 & 1 & \frac{5}{2} \end{array}\right] \xrightarrow{-3R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -2 & 0 & \frac{3}{2} \\ 0 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -\frac{1}{4} & -\frac{5}{8} \end{array}\right]$$

$$\xrightarrow[-R_3+R_2]{3R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & \frac{1}{8} \\ 0 & 0 & 1 & 1 & -\frac{1}{4} & -\frac{5}{8} \end{array}\right] = [I | (A^T)^{-1}]$$

$$\therefore (A^T)^{-1} = \begin{bmatrix} 1 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & \frac{1}{4} & \frac{1}{8} \\ 1 & -\frac{1}{4} & -\frac{5}{8} \end{bmatrix}$$

2. If $C = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$, find C^{-1}

$$\det(C) = \begin{vmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{vmatrix} = \begin{vmatrix} 2 & 0 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{vmatrix}$$

$$= 0$$

$\therefore$ C has NO inverse

i.e. C^{-1} does not exist.

Another way.

$$[C|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 5 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \cdots \sim \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & - & - & - \\ 0 & 1 & -1 & - & - & \\ 0 & 0 & 0 & - & & \end{array}\right]$$

$\therefore$ C^{-1} does not exist.