Math 260 – Quiz # 7c

Name: Solution Sec.#:________ Sr #:________________

1. Verify that $V = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is and eigenvector of the matrix $A = \begin{bmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{bmatrix}$.

$$AV = \begin{bmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = -2V$$

$\Rightarrow AV = -2V$

$\therefore$ V is an eigen vector associated with the eigenvalue $\lambda = -2$

2. Given that $\lambda = 2$, 2 and 3 are the eigenvalues of the matrix $B = \begin{bmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{bmatrix}$.

Is B diagonalizable?

We find the eigenvectors:

$\underline{\lambda = 2}$: $(B - 2I)X = 0 \Rightarrow \begin{bmatrix} 2 & -2 & 1 \\ 2 & -2 & 1 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} x_3 = t \\ x_2 = s \\ x_1 = -\frac{t}{2} + s \end{cases}$

$$X = \begin{bmatrix} -\frac{t}{2} + s \\ s \\ t \end{bmatrix} = \begin{bmatrix} -\frac{t}{2} \\ 0 \\ t \end{bmatrix} + \begin{bmatrix} s \\ s \\ 0 \end{bmatrix} = \frac{t}{2}\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ are the eigenvectors assocated with $\lambda = 2$

$\underline{\lambda = 3}$: $(B - 3I)X = 0 \Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 1 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \text{solving this system we get:} \\ x_1 = x_2 = x_3 \end{cases}$

$\therefore$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is the eigenvector associated with $\lambda = 3$.

Note that the Wronskian of the 3 eigenvectors is $\begin{vmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = -1 \neq 0$.
[i.e. they are linearly indep.]

So, B is diagonalizable.