Math 260 – Quiz # 7

Name: Solution Sec.#:_______ Sr #: ____________

Find the eigen values and the corresponding eigen vectors of $A = \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$.

Is A diagonalizable? if so, then find a diagonalizing matrix P a diagonal matrix D such that $P^{-1}AP = D$.

$$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 4-\lambda & -3 \\ 3 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 + 9 = 0$$

$\Rightarrow \lambda^2 - 8\lambda + 25 = 0 \Rightarrow \lambda = 4 \pm 3i$ are the eigen values.

$\Rightarrow$ A is diagonalizable.

To find P, we need to find the eigen vectors of A:

for $\underline{\lambda = 4+3i}$

$$[A-(4+3i)]X = 0 \Rightarrow \begin{bmatrix} -3i & -3 \\ 3 & -3i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & -3i & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow x - iy = 0 \Rightarrow y = 1, x = i \Rightarrow E_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}$$

Similarly, we have $E_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$.

$\therefore$ The eigenvectors are $E_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \Rightarrow P^{-1} = \frac{1}{2i} \begin{bmatrix} 1 & i \\ -1 & i \end{bmatrix} = \begin{bmatrix} \frac{1}{2i} & \frac{1}{2} \\ -\frac{1}{2i} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{-i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix}$$

$$D = \begin{bmatrix} 4+3i & 0 \\ 0 & 4-3i \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} \frac{-i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4+3i & 0 \\ 0 & 4-3i \end{bmatrix} = D$$