Math 202 Quiz # 3e

Name: Solution Sr. # ____________ Section # ____

Determine a function $M(x,y)$ such that the following DE is exact:

$$M dx + \underbrace{(xe^{xy} + 2xy + \frac{1}{x})}_{N} dy = 0$$

To be exact, we must have

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

But $\frac{\partial N}{\partial x} = xye^{xy} + e^{xy} + 2y - \frac{1}{x^2}$

$$= \frac{\partial M}{\partial y}$$

$$\therefore M = \int \left[xye^{xy} + e^{xy} + 2y - \frac{1}{x^2} \right] dy$$ ← see (*) below

$$= ye^{xy} - \cancel{\frac{e^{xy}}{x}} + \cancel{\frac{e^{xy}}{x}} + y^2 - \frac{1}{x^2} y + g(x)$$

$$\therefore M(x,y) = ye^{xy} + y^2 - \frac{y}{x^2} + g(x)$$

(*) $\int xye^{xy} dy$: Let $\begin{cases} u = y, & dv = xe^{xy} dy \\ du = dy, & v = e^{xy} \end{cases}$

$$\therefore \int xye^{xy} dy = \int y \, xe^{xy} dy = ye^{xy} - \int e^{xy} dy = ye^{xy} - \frac{1}{x} e^{xy}$$