Math 202 Quiz # 3d

Name: Solution Sr. # ____________ Section # ____

Verify that the following DE is exact and then solve the IVP:

$$\left(\frac{1}{1+y^2}+\cos x-2xy\right)\frac{dy}{dx}=y\,(y+\sin x),\qquad y(0)=1.$$

$$\underbrace{\left(y^2+y\sin x\right)}_{M}dx+\underbrace{\left(-\frac{1}{1+y^2}-\cos x+2xy\right)}_{N}dy=0$$

$$\frac{\partial M}{\partial y}=2y+\sin x \quad , \quad \frac{\partial N}{\partial x}=\sin x+2y$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \Rightarrow \text{Exact}$$

$$f(x,y)=\int\left(y^2+y\sin x\right)dx = xy^2-y\cos x+g(y)$$

$$\frac{\partial f}{\partial y}=2xy-\cos x+g'(y)=N=-\frac{1}{1+y^2}-\cos x+2xy$$

$$\Rightarrow g'(y)=-\frac{1}{1+y^2}$$

$$g(y)=-\int\frac{1}{1+y^2}dy = -\tan^{-1}y+K$$

$$\therefore f(x,y)=xy^2-y\cos x-\tan^{-1}y+K$$

The solution is $xy^2-y\cos x-\tan^{-1}y=C$

Now, $y(0)=1 \Rightarrow -1-\pi/4=C \Rightarrow C=-\dfrac{\pi+4}{4}$

$\therefore$ The solution is $xy^2-y\cos x-\tan^{-1}y=-\dfrac{\pi+4}{4}$