

Name: Solution Sr. # _____ Section # _____Verify that the following DE is exact and solve it $\frac{dy}{dx} = \frac{xy^2 - \sin x \cos x}{(1-x^2)y}$

$$\underbrace{(xy^2 - \sin x \cos x)}_M dx + \underbrace{(x^2 - 1)y}_{N} dy = 0$$

$$\frac{\partial M}{\partial y} = 2xy, \quad \frac{\partial N}{\partial x} = 2xy$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{Exact.}$$

$$f(x, y) = \int (xy^2 - \sin x \cos x) dx$$

$$= \frac{1}{2} x^2 y^2 - \frac{1}{2} \sin^2 x + g(y)$$

$$\frac{\partial f}{\partial y} = x^2 y + g'(y) = N = (x^2 - 1)y = x^2 y - y$$

$$\Rightarrow g'(y) = -y$$

$$g(y) = -\frac{1}{2} y^2 + K$$

$$f(x, y) = \frac{1}{2} x^2 y^2 - \frac{1}{2} \sin^2 x - \frac{1}{2} y^2 + K$$

The solution is

$$\frac{1}{2} x^2 y^2 - \frac{1}{2} \sin^2 x - \frac{1}{2} y^2 = C_1$$

$$\text{i.e. } x^2 y^2 - \sin^2 x - y^2 = C$$