Math 202 Quiz # 3f

Name: *Solution* Sr. # ____________ Section # ____

Find an integrating factor that will convert the following DE into exact and solve it:

$$\frac{dy}{dx} = \frac{xy}{20-2x^2-3y^2}$$

$$\underbrace{xy}_{M}\,dx + \underbrace{(3y^2+2x^2-20)}_{N}\,dy = 0 \qquad \text{———} \quad (1)$$

$$\frac{\partial M}{\partial y} = x \;, \quad \frac{\partial N}{\partial x} = 4x \;\Rightarrow\; \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow \text{not exact}$$

$$\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{1}{3y^2+2x^2-20}\left[x-4x\right] = \frac{-3x}{3y^2+2x^2-20}$$

is not a function of x only [depends on x and y], so we find:

$$\frac{1}{M}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{1}{xy}\left[-3x\right] = -\frac{3}{y} = g(y)$$

The integrating factor is $\mu(y) = e^{-\int g(y)dy} = e^{\int \frac{3}{y}dy} = e^{3\ln y} = y^3$

Multiply (1) by y^3:

$$\underbrace{xy^4}_{M}\,dx + \underbrace{(3y^5+2x^2y^3-20y^3)}_{N}\,dy = 0 \quad \text{[Exact]}$$

$$f(x,y) = \int xy^4\,dx = \tfrac{1}{2}x^2y^4 + g(y)$$

$$\frac{\partial f}{\partial y} = 2x^2y^3 + g'(y) = N = 3y^5+2x^2y^3-20y^3 \Rightarrow g'(y) = 3y^5 - 20y^3$$

$$\Rightarrow g(y) = \tfrac{1}{2}y^6 - 5y^4 + K$$

$$\therefore f(x,y) = \tfrac{1}{2}x^2y^4 + \tfrac{1}{2}y^6 - 5y^4 + K$$

The solution is $\tfrac{1}{2}x^2y^4 + \tfrac{1}{2}y^6 - 5y^4 = C$