Math 202 Quiz # 3

Name: Solution Sr. # ________ Section # ____

Verify that the following DE is exact and solve it : $\dfrac{dy}{dx} = \dfrac{y\cos xy - e^{2y}}{2xe^{2y} - x\cos xy + 2y}$

$$\underbrace{\left(y\cos xy - e^{2y}\right)}_{M} dx + \underbrace{\left(-2xe^{2y} + x\cos xy - 2y\right)}_{N} dy = 0$$

$$\left.\begin{array}{l} \dfrac{\partial M}{\partial y} = -xy\sin xy + \cos xy - 2e^{2y} \\[2ex] \dfrac{\partial N}{\partial x} = -2e^{2y} - xy\sin xy + \cos xy \end{array}\right\} \Rightarrow \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} \Rightarrow \text{Exact}$$

$$f(x,y) = \int \left(y\cos xy - e^{2y}\right) dx$$

$$= \sin xy - xe^{2y} + g(y)$$

$$\frac{\partial f}{\partial y} = x\cos xy - 2xe^{2y} + g'(y) = N = -2xe^{2y} + x\cos xy - 2y$$

$$\Rightarrow \quad g'(y) = -2y$$

$$g(y) = -y^2 + K$$

$$\therefore f(x,y) = \sin xy - xe^{2y} - y^2 + K$$

The solution is

$$\sin xy - xe^{2y} - y^2 = C$$