Questions for review on Math 202

Elements of Differential Equations

Prepared by

Mohammad Samman

Department of Mathematics and Statistics
KFUPM

May 2008

1. State what is meant by Differential Equations.

2. Do you know any application for Differential Equations; give some examples.

3. Write a brief classification with examples of the types of DEs that you studied in your course Math 202.

4. Does every differential equation have a solution.

5. If we know a solution for a given DE, is it necessarily to be unique?

6. What do we mean by an initial value Problem?

7. What do we mean by Cauchy-Euler differential equation? Give an example and show how to solve such type of equations.

8. Complete the following table

Equation	Order	Linear / Nonlinear
$y' = 10 + y^2$		
$x^2\,dy + 5xy\,dx = 0$		
$y = 2xy' + y\,(y')^2$		
$y'' + y = \tan x$		
$y'' - 5y' + 6y = 0$		
$y' + 3x(y'')^3 = \sin x$		
$y' + 3\sin x\, y'' = \cos x$		

9. **Classify** the following 1st Order ODE as **Separable**, **Linear in *y*** (or **in *x***), **Homogeneous** (**with its degree**), **Bernoulli**, or **Exact**.

   i. $(y + y^2)dx - (x + x^2)dy = 0$

   ii. $(y - xy^2)dy = ydx$

   iii. $(e^{y/x} + e^{x^3/y^3} + 1)dy = (1 + \ln(y/x))dx$

   iv. $\frac{dy}{dx} = \sqrt{x^2 - y^2}$

   v. $3\frac{dy}{dx} = 4x - y$

10. Solve $x^2 \frac{dy}{dx} = y - xy$

11. Solve $x\frac{dy}{dx} - y = x^2 \sin x$

12. Solve the initial value problem $(e^x + y)dx + (2 + x + ye^y)dy = 0,\ \ y(0) = 1.$

13. Solve the initial value problem $\frac{dy}{dx} = \cos(x+y),\ y(0) = \pi/4$

14. Solve $x\frac{dy}{dx} - (1+x)y = xy^2$

15. Solve $(y^2 - xy)dx + x^2 dy = 0$

16. Is $y = xe^{-2x}$ a solution to $y'' + 4y' + 4y = 0$?

17. How many solutions are there to the initial value problem $\frac{1}{x^2}\frac{dy}{dx} + y^2 = \frac{1}{x},\qquad y(0) = 2$. Justify your answer.

18. The Population of a Community is known to increase at a rate Proportional to the number of People present at any time. The Population of the community is doubled after 5 years and it is10,000 after 3 years. What was the initial population. What will be the Population after 10 years.

19. If $y_1 = \ln x$ is a solution of the equation $xy'' + y' = 0$, use **reduction of order** Or an appropriate formula to find a second solution.

20. Solve the boundary value problem: $y'' - 10y' + 25y = 0,, y(0) = 1,\ \ y(1) = 0.$

21. Find the general solution of the following **Cauchy-Euler Equation**

$$2x^2 y'' + 5xy' + y = 0$$

22. Find the solution of the BVP $y^{(4)} + y'' = 0$ satisfying the conditions:

$$y(0) = 0,\ y(\pi) = 0,\ y'(0) = 1,\ y'(\pi) = -1$$

23. Write a homogeneous linear differential equation whose auxiliary equation is

$$5m^5 - 2m^3 + 4m = 0$$

24. Given $y_1 = x \sin(linx)$ a solution of the DE $x^2 y'' - xy' + 2y = 0$. Find another solution for this equation.

25. Using Wronskian show that the functions 1, $1/x$ and $\log x$ are linearly independent on the interval $(0, \infty)$.

26. Show that 1, x, $\sin x$, $\cos x$ form **a Fundamental Set of the solutions** of the Differential Equation $y^{(4)} + y'' = 0$ on $(-\infty, \infty)$.

27. Use the method of **Variation of Parameters** to find the general solution of the differential equation $\frac{d^2y}{dx^2} + y = \sin x$

28. Solve the above question using the method of **Undetermined Coefficients**.

29. Solve the DE: $y''' - xy'' = 8x^2$.

30. If $y_p = u_1y_1 + u_2y_2 + u_3y_3$ is a particular solution of $y^{(3)} + 9y^{(1)} = \tan x$, then find: (i) y_1, y_2, y_3 (ii) u_1', u_2', u_3'

31. Find all Singular Points of the ODE and classify them as regular or irregular singular point: $x^3(x^2 - 9)y'' - 2x^2(x+3)y' + (x-3)y = 0$

32. Use the **Power Series method** to find the General solution of the DE

$$y'' - 4xy' - 4y = e^x \qquad \text{about } x_0 = 0.$$

33. Show that $x_0 = 0$ is a regular singular point of the differential equation

$$(6x + 2x^3)y'' + 21xy' + 9(x^2 - 1)y = 0 .$$

Then find the **Indicial Equation** and its roots about $x_0 = 0$.

34. Use **Gauss-Jordan Elimination Method**, to solve the system

$$\begin{aligned} s - t + u + v &= 0 \\ 2s + 2u &= 0 \\ s + t + u - v &= 0 \\ -s - 3t - u + 3v &= 0 \end{aligned}$$

35. Find the inverse of A, if it exists, where $A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$

36. Find the **eigen values** of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, and find the corresponding **eigen vectors**.

37. Solve the system

$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = 2x + 2y - z$$
$$\frac{dz}{dt} = y$$

38. Solve the system

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = -4x + 3y$$

39. Solve the system

$$X' = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 0 \\ 6 & 3 & -8 \end{bmatrix} X$$

40. Solve the following non homogeneous system

$$X' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ te^t \\ e^t \end{bmatrix}$$

41. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}$.

Compute e^{At} and then use it to find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X .$$