

Math 260 – Quiz # 7

Name: Solution

Sr #: _____

The Augmented matrix of a non-homogeneous system $AX = B$ of linear equations has the following Reduced Form:

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]. \text{ Find a basis for the solution space of } AX = 0.$$

$$AX = 0 \Rightarrow \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_4 + x_5 = 0 \Rightarrow x_4 = -x_5$$

Take $x_5 = r$ then $x_4 = -r$

$x_1 = x_5 - x_3 - 2x_2$. Take $x_3 = t$ and $x_2 = s$. then

$$x_1 = r - t - 2s$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} r - t - 2s \\ s \\ t \\ -r \\ r \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ 0 \\ -r \\ r \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2s \\ s \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= r \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

\therefore A basis for the solution space is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.