Math 260 – Quiz # 9

Name: Solution Sr #: ____________

Use the method of **variation of parameters** to solve the DE: $y'' + y = \sec x \tan x$ — $(*)$

First we solve the associated hom. eq. $y'' + y = 0$ — (1)

$\Rightarrow \lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$

$\therefore$ The solution for (1) is $y_h = c_1 \cos x + c_2 \sin x$ — (2)

So, we have $y_1 = \cos x$, $y_2 = \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ +\sec x \tan x & \cos x \end{vmatrix} = -\sin x \sec x \tan x = -\tan^2 x$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \tan x \end{vmatrix} = \cos x \sec x \tan x = \tan x$$

$$u_1' = \frac{W_1}{W} = -\tan^2 x, \quad u_2' = \frac{W_2}{W} = \tan x$$

$$u_1 = \int -\tan^2 x \, dx = -\int (\sec^2 x - 1)\,dx = \int (1 - \sec^2 x)\,dx = x - \tan x$$

$$u_2 = \int \tan x \, dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| = \ln|\sec x|$$

$\therefore$ a particular solution for $(*)$ is: $y_p = u_1 \cos x + u_2 \sin x$

$$\Rightarrow y_p = (x - \tan x)\cos x + \ln|\sec x| \sin x$$
$$= x\cos x - \sin x + \sin x \ln|\sec x|$$

$\therefore$ The solution for $(*)$ is $y = y_h + y_p$

$$= c_1 \cos x + c_2 \sin x + x\cos x + \sin x \ln|\sec x|$$