Math 202 Quiz # 9

Name: Solution Section # __________ Sr. # __________

Solve the initial value problem $X' = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) = 0 \Rightarrow \lambda = 1, 2$$

$\underline{\lambda = 1}$

$$(A - I)K_1 = 0 \Rightarrow \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \Rightarrow k_2 = 0, k_1 = c. \text{ Take } k_1 = 1$$

$$K_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t}$$

$\underline{\lambda = 2}$

$$(A - 2I)K_2 = 0 \Rightarrow \left[\begin{array}{cc|c} -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow k_1 = 0, \ k_2 = 1.$$

$$K_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$$

The general solution is $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$

$$X(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \Rightarrow c_1 = 2, \ c_2 = 1$$

$\therefore$ The solution for the given IVP is

$$X = \begin{bmatrix} 2 \\ 0 \end{bmatrix} e^{t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$$