Math 202 Quiz # 7

Name: Solution Section # ______ Serial # ____

1. Verify that $X_p = \begin{bmatrix} 2 \\ -1 \end{bmatrix} t + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ is a particular solution of $X' = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} X + \begin{bmatrix} 2t-7 \\ -4t-18 \end{bmatrix}$.

$$X_p = \begin{bmatrix} 2t+5 \\ -t+1 \end{bmatrix}$$

$$\Rightarrow X_p' = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} X_p + \begin{bmatrix} 2t-7 \\ -4t-18 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2t+5 \\ -t+1 \end{bmatrix} + \begin{bmatrix} 2t-7 \\ -4t-18 \end{bmatrix}$$

$$= \begin{bmatrix} -2t+9 \\ 4t+17 \end{bmatrix} + \begin{bmatrix} 2t-7 \\ -4t-18 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix} = X_p'$$

Hence

$$X_p' = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} X_p + \begin{bmatrix} 2t-7 \\ -4t-18 \end{bmatrix}$$

i.e. X_p is a particular solution of the given system.

2. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & -2 & 2 \\ -2 & 1-\lambda & -2 \\ 2 & -2 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)^2 - 4\right] + 2\left[-2(1-\lambda) + 4\right] + 2\left\{4 - 2(1-\lambda)\right\} = 0$$

$$(1-\lambda)\left[\lambda^2 - 2\lambda - 3\right] + 2\left[2\lambda + 2\right] + 2\left[2\lambda + 2\right] = 0$$

$$(1-\lambda)(\lambda-3)(\lambda+1) + 8(\lambda+1) = 0$$

$$(\lambda+1)\left[(1-\lambda)(\lambda-3) + 8\right] = 0$$

$$(\lambda+1)(-\lambda^2 + 4\lambda + 5) = 0$$

$$(\lambda+1)(\lambda+1)(-\lambda+5) = 0$$

$$(\lambda+1)^2(-\lambda+5) = 0 \Longrightarrow \lambda = -1, -1, 5$$

<u>for $\lambda = -1$</u>

$$(\lambda + I)K = 0 \Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ -2 & 2 & -2 & 0 \\ 2 & -2 & 2 & 0 \end{array}\right] \xrightarrow[\text{operations}]{\text{row}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$k_1 = k_2 - k_3$

$k_2 = 0, k_3 = 1 \Rightarrow k_1 = -1 \Rightarrow K_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$k_2 = 1, k_3 = 0 \Rightarrow k_1 = 1 \Rightarrow K_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

<u>for $\lambda = 5$</u>

$$(A + 5I)K = 0 \Rightarrow \left[\begin{array}{ccc|c} -4 & -2 & 2 & 0 \\ -2 & -4 & -2 & 0 \\ 2 & -2 & -4 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} k_2 = -k_3 \\ k_1 = k_3 \end{array}$$

$k_3 = 1 \Rightarrow \quad \therefore K_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\therefore$ the eigenvectors are: $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$