Math 202 Quiz # 5

Name: Solution Section # ___________ Serial # ____

Solve the following DE by using two different methods (one is by reducing it to a DE with constant coefficients)

$$x^2y''+3xy'+y=0$$

[This is Cauchy-Euler eqn]

Method 1: Put $y=x^m \Rightarrow y'=mx^{m-1}$, $y''=m(m-1)x^{m-2}$

Substitute $\Rightarrow$ $x^2m(m-1)x^{m-2}+3xmx^{m-1}+x^m=0$

$m(m-1)x^m+3mx^m+x^m=0$

$[m(m-1)+3m+1]x^m=0 \Rightarrow m^2+2m+1=0 \Rightarrow m=-1,-1$

$\therefore$ the solution is $\boxed{y=c_1x^{-1}+c_2x^{-1}\ln x}$

Method 2: put $x=e^t$. Then $t=\ln x$ and

$$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{1}{x}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2}=\underbrace{\frac{d}{dx}\left(\frac{dy}{dx}\right)=\ \cdots\cdots}_{\text{See your notes}}=\frac{1}{x^2}\left[\frac{d^2y}{dt^2}-\frac{dy}{dt}\right]$$

Substitute $\Rightarrow x^2\left\{\frac{1}{x^2}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)\right\}+3x\left\{\frac{1}{x}\frac{dy}{dt}\right\}+y=0$

$\frac{d^2y}{dt^2}-\frac{dy}{dt}+3\frac{dy}{dt}+y=0 \Rightarrow y''+2y'+y=0$, where $y'=\frac{dy}{dt}$

$\lambda^2+2\lambda+1=0 \Rightarrow \lambda=-1,-1 \Rightarrow y=c_1e^{-t}+c_2te^{-t}$

But $x=e^t \Longrightarrow$ the solution is

$$\boxed{y=c_1x^{-1}+c_2x^{-1}\ln x}$$