

Questions for review on Math 202

Elements of Differential Equations

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May 2008

1. State what is meant by Differential Equations.
2. Do you know any application for Differential Equations; give some examples.
3. Write a brief classification with examples of the types of DEs that you studied in your course Math 202.
4. Does every differential equation have a solution.
5. If we know a solution for a given DE, is it necessarily to be unique?
6. What do we mean by an initial value Problem?
7. What do we mean by Cauchy-Euler differential equation? Give an example and show how to solve such type of equations.
8. Complete the following table

Equation	Order	Linear / Nonlinear
$y' = 10 + y^2$		
$x^2 dy + 5xy dx = 0$		
$y = 2xy' + y (y')^2$		
$y'' + y = \tan x$		
$y'' - 5y' + 6y = 0$		
$y' + 3x(y'')^3 = \sin x$		
$y' + 3\sin x y'' = \cos x$		

9. **Classify** the following 1st Order ODE as **Separable**, **Linear in y** (or **in x**), **Homogeneous** (with its degree), **Bernoulli**, or **Exact**.

i. $(y + y^2)dx - (x + x^2)dy = 0$

ii. $(y - xy^2)dy = ydx$

iii. $(e^{y/x} + e^{x^3/y^3} + 1)dy = (1 + \ln(y/x))dx$

iv. $\frac{dy}{dx} = \sqrt{x^2 - y^2}$

v. $3\frac{dy}{dx} = 4x - y$

10. Solve $x^2 \frac{dy}{dx} = y - xy$
11. Solve $x \frac{dy}{dx} - y = x^2 \sin x$
12. Solve the initial value problem $(e^x + y)dx + (2 + x + ye^y)dy = 0$, $y(0) = 1$.
13. Solve the initial value problem $\frac{dy}{dx} = \cos(x + y)$, $y(0) = \pi/4$
14. Solve $x \frac{dy}{dx} - (1 + x)y = xy^2$
15. Solve $(y^2 - xy)dx + x^2 dy = 0$
16. Is $y = xe^{-2x}$ a solution to $y'' + 4y' + 4y = 0$?
17. How many solutions are there to the initial value problem $\frac{1}{x^2} \frac{dy}{dx} + y^2 = \frac{1}{x}$, $y(0) = 2$. Justify your answer.
18. The Population of a Community is known to increase at a rate Proportional to the number of People present at any time. The Population of the community is doubled after 5 years and it is 10,000 after 3 years. What was the initial population. What will be the Population after 10 years.
19. If $y_1 = \ln x$ is a solution of the equation $xy'' + y' = 0$, use **reduction of order** Or an appropriate formula to find a second solution.
20. Solve the boundary value problem: $y'' - 10y' + 25y = 0$, $y(0) = 1$, $y(1) = 0$.
21. Find the general solution of the following **Cauchy-Euler Equation**
 $2x^2 y'' + 5xy' + y = 0$
22. Find the solution of the BVP $y^{(4)} + y'' = 0$ satisfying the conditions:
 $y(0) = 0$, $y(\pi) = 0$, $y'(0) = 1$, $y'(\pi) = -1$
23. Write a homogeneous linear differential equation whose auxiliary equation is
 $5m^5 - 2m^3 + 4m = 0$
24. Given $y_1 = x \sin(\ln x)$ a solution of the DE $x^2 y'' - xy' + 2y = 0$. Find another solution for this equation.

25. Using Wronskian show that the functions 1 , $1/x$ and $\log x$ are linearly independent on the interval $(0, \infty)$.
26. Show that 1 , x , $\sin x$, $\cos x$ form a **Fundamental Set of the solutions** of the Differential Equation $y^{(4)} + y'' = 0$ on $(-\infty, \infty)$.
27. Use the method of **Variation of Parameters** to find the general solution of the differential equation $\frac{d^2 y}{dx^2} + y = \sin x$
28. Solve the above question using the method of **Undetermined Coefficients**.
29. Solve the DE: $y''' - xy'' = 8x^2$.
30. If $y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$ is a particular solution of $y^{(3)} + 9y^{(1)} = \tan x$, then find: (i) y_1, y_2, y_3 (ii) u_1', u_2', u_3'
31. Find all Singular Points of the ODE and classify them as regular or irregular singular point: $x^3(x^2 - 9)y'' - 2x^2(x + 3)y' + (x - 3)y = 0$
32. Use the **Power Series method** to find the General solution of the DE $y'' - 4xy' - 4y = e^x$ about $x_0 = 0$.
33. Show that $x_0 = 0$ is a regular singular point of the differential equation $(6x + 2x^3)y'' + 21xy' + 9(x^2 - 1)y = 0$. Then find the **Indicial Equation** and its roots about $x_0 = 0$.
34. Use **Gauss-Jordan Elimination Method**, to solve the system

$$\begin{aligned} s - t + u + v &= 0 \\ 2s + 2u &= 0 \\ s + t + u - v &= 0 \\ -s - 3t - u + 3v &= 0 \end{aligned}$$

35. Find the inverse of A , if it exists, where $A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$

36. Find the **eigen values** of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, and find the corresponding **eigen vectors**.

37. Solve the system

$$\frac{dx}{dt} = x$$

$$\frac{dy}{dt} = 2x + 2y - z$$

$$\frac{dz}{dt} = y$$

38. Solve the system

$$\frac{dx}{dt} = 3x + 4y$$

$$\frac{dy}{dt} = -4x + 3y$$

39. Solve the system

$$X' = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 0 \\ 6 & 3 & -8 \end{bmatrix} X$$

40. Solve the following non homogeneous system

$$X' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ te^t \\ e^t \end{bmatrix}$$