KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Department of Mathematics and Statistics

MATH 550-(071)

Take home Exam II

Due on 12 January, 2008

Name: Sec.# I.D. #

Show All Necessary Work

Question	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

1. Determine whether the following matrix is diagonalizable over *R*:

$$\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$$

2. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and let $T \in L(R^2, R^2)$, which is represented by the matrix A in the standard ordered bases for R^2, and let $U \in L(C^2, C^2)$, which is also represented by A in the standard basis.

(a) Find the characteristic polynomial for T and for U.

(b) Find the characteristic values for T and for U, and for each such characteristic value c find a basis for the corresponding space of characteristic vectors.

3. (a) Show that similar matrices have the same trace.

(b) If A and B are $n \times n$ matrices over C, show that $AB - BA \neq I$

4. Let $A = \begin{bmatrix} c & 1 & 0 \\ 0 & c & 1 \\ 0 & 0 & c \end{bmatrix}$. Find the characteristic polynomial and the minimal polynomial of A.

5. Let $\alpha_1 = (1, 2, -3, 4), \alpha_2 = (0, 1, 4, -1)$, and let $W = span(\{\alpha_1, \alpha_2\})$. Find a basis for the annihilator space W°.

6. Consider $V = M_n(R)$, the space of $n \times n$ matrices over the field of real numbers. Let A be a fixed matrix in V, and let $T_A, U_A \in L(V, V)$, where $T_A(B) = AB$, and $U_A(B) = AB - BA$.

   (a) Show that if A is diagonalizable then T_A and U_A are diagonalizable.

   (b) Let Ω be the family of linear operators T_A obtained by letting A vary over all diagonal matrices. Show that the operators in Ω are simultaneously diagonalizable.

7. Suppose T is an invertible linear operator. Show that if c is an eigen value of T then c^{-1} is an eigen value of T^{-1}.

8. Let A be a $n \times n$ matrix with real entries. Prove that, if A is not similar over R to a triangular matrix, then A is similar over C to a diagonal matrix.

9. Let V be the space of all polynomial functions over the field of real numbers, and let a and b be fixed real numbers. Let f be the linear functional on V defined by $f(p) = \int_a^b p(x)dx$. Consider the differential operator D on V. Find $D^t f$? (i.e. the transpose of D).

10. Show that every idempotent matrix is diagonalizable.
(A square matrix A is idempotent if $A^2 = A$)