Math 260 Quiz # 2

Name: Solution Serial # ______

1. Solve the initial value problem: $(x^2+4)y' + 3xy = x \quad ; y(0) = 2$

$$(x^2+4)\frac{dy}{dx} + 3xy = x$$

$$\frac{dy}{dx} + \frac{3x}{x^2+4}y = \frac{x}{x^2+4} \qquad (\text{linear eq}^n)$$

$$\rho(x) = e^{\int \frac{3x}{x^2+4}dx} = e^{\frac{3}{2}\int \frac{2x}{x^2+4}dx} = e^{\frac{3}{2}\ln(x^2+4)} = (x^2+4)^{3/2}$$

$$\frac{d}{dx}\left[y(x^2+4)^{3/2}\right] = \frac{x}{x^2+4}(x^2+4)^{3/2} = x(x^2+4)^{\frac{1}{2}}$$

$$y(x^2+4)^{3/2} = \int x(x^2+4)^{\frac{1}{2}}dx = \frac{1}{2}\int (x^2+4)^{\frac{1}{2}}(2x)dx = \frac{1}{3}(x^2+4)^{3/2} + C$$

$$y(x) = \frac{1}{3} + C(x^2+4)^{-3/2}$$

$$y(0) = 2 \Rightarrow \frac{1}{3} + C(4)^{-3/2} = 2 \Rightarrow C = \frac{40}{3}$$

$$\therefore y(x) = \frac{1}{3} + \frac{40}{3}(x^2+4)^{-3/2}$$

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2. Put the following DE in the standard form as a linear DE, regarding y as the independent variable rather than x. Then solve it: $(1+2xy)\frac{dy}{dx} = 1+y^2$

$$\frac{dy}{dx} = \frac{1+y^2}{1+2xy} \Rightarrow \frac{dx}{dy} = \frac{1+2xy}{1+y^2}$$

$$\frac{dx}{dy} = \frac{1}{1+y^2} + \frac{2xy}{1+y^2} \Rightarrow \frac{dx}{dy} - \frac{2xy}{1+y^2} = \frac{1}{1+y^2}$$

$$\frac{dx}{dy} + \left(\frac{-2y}{1+y^2}\right)x = \frac{1}{1+y^2} \qquad (\text{linear})$$

$$\rho(y) = e^{\int \frac{-2y}{1+y^2}dy} = e^{-\ln(1+y^2)} = \frac{1}{1+y^2}$$

$$\frac{d}{dy}\left[\frac{1}{1+y^2}x\right] = \frac{1}{(1+y^2)^2}$$

$$\frac{1}{1+y^2}x = \int \frac{1}{(1+y^2)^2}dy = \frac{1}{2}\tan^{-1}y + \frac{y}{2(1+y^2)} + C \quad [\text{How?}]$$

To evaluate $\int \frac{1}{(1+y^2)^2}dy$ use the substitution $y = \tan u$ ----

$$x = \left(\frac{1+y^2}{2}\right)\tan^{-1}y + \frac{y}{2} + C(1+y^2)$$