Math 260 Quiz # 1

Name: Solution I.D. # ____________ Serial # ____

1. Verify that the function $y(x) = \ln(x + c)$ is a solution for the differential equation $e^y y' = 1$. Then find the constant c which satisfies the initial condition $y(0) = 0$.

$$y = \ln(x+c) \Rightarrow y' = \frac{1}{x+c}$$

substitute in the DE,

$$e^y y' = e^{\ln(x+c)} \cdot \frac{1}{x+c} = (x+c)\frac{1}{x+c} = 1$$

$\therefore$ $y(x)$ satisfies the given DE and so $y(x)$ is a solution.

$$y(0) = 0 \Rightarrow \ln(c) = 0 \Rightarrow c = 1.$$

$\therefore$ the solution for the IVP is $y(x) = \ln(x+1)$.

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2. Solve the initial value problem: $\dfrac{dy}{dx} = x\sqrt{x^2+9}$; $y(-4) = 0$

$$y(x) = \int x\sqrt{x^2+9}\, dx$$

$$= \frac{1}{2}\int (x^2+9)^{\frac{1}{2}}\, 2x\, dx$$

$$= \frac{1}{3}(x^2+9)^{\frac{3}{2}} + C$$

Since $y(-4) = 0 \Rightarrow y(-4) = \frac{1}{3}(16+9)^{\frac{3}{2}} + C = 0$

$$\Rightarrow C = -\frac{1}{3}(25)^{\frac{3}{2}} = -\frac{125}{3}$$

$\therefore$ the solution for the IVP is

$$y = \frac{1}{3}(x^2+9)^{\frac{3}{2}} - \frac{125}{3}$$