Name: _____Solution_____   Serial # _____

1. Write the vector $h = (2, -7, 9)$ as a linear combination of the vectors u, v, and w where $u = (1, -1, 2)$, $v = (3, 0, 1)$, $w = (1, -2, 2)$.

$$h = c_1 u + c_2 v + c_3 w$$

$$(2, -7, 9) = c_1(1, -1, 2) + c_2(3, 0, 1) + c_3(1, -2, 2)$$

$$\Rightarrow \quad \begin{array}{rcl} c_1 + 3c_2 + c_3 &=& 2 \\ -c_1 \qquad\quad -2c_3 &=& -7 \\ 2c_1 + c_2 + 2c_3 &=& 9 \end{array} \Bigg\}$$

Solving this system for c_1, c_2, c_3:

$$\Rightarrow c_1 = 3$$
$$c_2 = -1$$
$$c_3 = 2$$

$$\therefore \quad h = 3u - v + 2w$$

2. Show that the set of all (x, y, z) such that $x = y + z$ is a subspace of R^3.

Let $W = \{(x, y, z) : x = y + z\}$. We show that W is a subspace.

Let $u, v \in W$. So, $u = (y_1 + z_1, y_1, z_1)$, $v = (y_2 + z_2, y_2, z_2)$.

Then $u + v = (y_1 + z_1, y_1, z_1) + (y_2 + z_2, y_2, z_2)$

$$= (y_1 + z_1 + y_2 + z_2, y_1 + y_2, z_1 + z_2)$$

$$= ((y_1 + y_2) + (z_1 + z_2), y_1 + y_2, z_1 + z_2) \in W$$

$$\Rightarrow u + v \in W.$$

Also, for any $c \in R$,

$$cu = c(y_1 + z_1, y_1, z_1) = (c(y_1 + z_1), cy_1, cz_1)$$

$$= (cy_1 + cz_1, cy_1, cz_1) \in W$$

$$\Rightarrow cu \in W.$$

Hence W is a subspace of R^3.