Math 260 Quiz # 4

Name: Solution Serial # ______

1. Write the vector $h = (2,-7,9)$ as a linear combination of the vectors u, v, and w where $u = (1,-2,2)$, $v = (3,0,1)$, $w = (1,-1,2)$.

$$h = c_1 u + c_2 v + c_3 w$$

$$(2,-7,9) = c_1(1,-2,2) + c_2(3,0,1) + c_3(1,-1,2)$$

$$\Rightarrow \left.\begin{aligned} c_1 + 3c_2 + c_3 &= 2 \\ -2c_1 \quad - c_3 &= -7 \\ 2c_1 + c_2 + 2c_3 &= 9 \end{aligned}\right\} \text{ Solving this system for } c_1, c_2, c_3:$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ -2 & 0 & -1 & -7 \\ 2 & 1 & 2 & 9 \end{array}\right] \rightsquigarrow \cdots \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \begin{aligned} c_1 &= 2 \\ c_2 &= -1 \\ c_3 &= 3 \end{aligned}$$

$$\therefore \; h = 2u - v + 3w$$

2. Show that the set of all (x,y,z) such that $z = x + y$ is a subspace of R^3.

Let $W = \{(x,y,z) : z = x+y\}$. We show that W is a subspace.

Let $u, v \in W$. So $u = (x_1, y_1, x_1+y_1)$, $v = (x_2, y_2, x_2+y_2)$.
Then

$$\begin{aligned} u+v &= (x_1, y_1, x_1+y_1) + (x_2, y_2, x_2+y_2) \\ &= (x_1+x_2, y_1+y_2, x_1+y_1+x_2+y_2) \\ &= (x_1+x_2, y_1+y_2, (x_1+x_2)+(y_1+y_2)) \in W \end{aligned}$$

So, $u+v \in W$.

Also, for any $c \in \mathbb{R}$,

$$\begin{aligned} cu = c(x_1, y_1, x_1+y_1) &= (cx_1, cy_1, c(x_1+y_1)) \\ &= (cx_1, cy_1, cx_1+cy_1) \in W \end{aligned}$$

So, $cu \in W$. Hence W is a subspace of $\mathbb{R}^3$.